8.3 Antiderivatives

NOTES

CALCULUS

Write your questions here!

Find the antiderivative of the following.

$f'(x) = \sqrt[5]{x^2} + \dfrac{4}{\sqrt[4]{x^3}}$

$f'(x) = \dfrac{1}{x}$	$y' = e^x$	$\dfrac{dy}{dx} = \cos x$	$\dfrac{d}{dx}y = \sin x$

INITIAL CONDITION

Find the function f that satisfies the given condition

$f'(x) = 3x^2 - x + 4$ and $f(2) = 8$	$f'(x) = 3\sqrt{x} + 3$ and $f(1) = 4$

Find y, if $y'' = \cos x$ and $y'\left(\frac{\pi}{2}\right) = 2$ and $y\left(\frac{\pi}{2}\right) = 3\pi$

Particle Motion

A particle moves along the x-axis with an acceleration of $a(t) = 12t - 4$. The particle's velocity is 18 centimeters per second at $t = 2$. The initial position of the particle is 8 cm. What is the position of the particle at $t = 3$?

SUMMARY:

Now, summarize your notes here!

Find the antiderivatives of the following.

1. $f'(x) = 4\sqrt[3]{x^2} - \frac{5}{\sqrt[5]{x^3}} + 2$

2. $\frac{dy}{dx} = x^{-2} - x^{-1}$

3. $y' = \sin x + x^{\frac{3}{2}}$

Evaluate the indefinite integrals.

4. $\int (3x + e^x)\, dx$

5. $\int \left(3e^x + \frac{9}{x}\right) dx$

6. $\int (4 - \cos x)\, dx$

Evaluate the definite integrals using the Fundamental Theorem of Calculus.

7. $$\int_{\pi}^{\frac{3\pi}{2}} (2 + \cos x)dx$$

8. $$\int_0^1 (e^x - x)dx$$

Find the function that satisfies the given conditions.

9. $h'(t) = 8t^3 + 5$ and $h(1) = -4$

10. $\frac{dy}{dx} = 2x + \sin x$ and $y(0) = 4$

Find the function that satisfies the given conditions.

11. $f''(x) = x^{-3/2}$ and $f'(4) = 2$ and $f(0) = 0$

12. $f''(x) = \sin x$ and $f'(0) = 1$ and $f(0) = 6$

Word Problems!

13. A particle moves along the y-axis with an acceleration of $a(t) = 2$ where t is time in seconds. The particle's velocity at $t = 2$ is 5 cm/sec. The position of the function at $t = 2$ is 10cm. What is the position of the particle at $t = 6$?

14. A ball is thrown down off of a house with a velocity of $v(t) = -32t - 8$ where t is time in seconds and v is ft/sec. The ball is 20 feet in the air at $t = 1$. What is the initial height of the ball?

15. A particle moves along the y-axis with an acceleration of $a(t) = 12t - 4$ with initial velocity of -10 and initial position 0. Find the position of the function at the particle's minimum velocity.

16. The graph of f includes the point (2,6) and the slope of the tangent line to f at any point x is given by the expression $3x + 4$. Find $f(-2)$.

Word Problems!	
17. A coin is dropped from a 850 foot building The velocity of the coin is $v(t) = -16t$. Find the both the position function and acceleration function.	18. A particle moves along the *x*-axis with a velocity of $v(t) = \sqrt[3]{t^2} - \frac{1}{t^2}$ measured in inches/second. At $t = 1$ the position of the particle is 3 inches. What is the particle's position at $t = 8$?
19. A particle moves along the *x*-axis with a velocity of $v(t) = 1 - \sin t$. At $t = \pi$ seconds the position of the particle is π inches. What is the position of the particle at $t = \frac{3\pi}{2}$?	20. A particle moves along the *y*-axis with a velocity of $v(t) = \frac{1}{t} - \frac{t^2}{3} + 2$. At $t = 1$ seconds the position of the particle is 8 meters. Find the both the acceleration and position function.

8.3 Antiderivatives

TEST PREP

MULTIPLE CHOICE

1. If $f'(x) = 12x^2 - 6x + 1, f(1) = 5,$ then $f(0)$ equals
   - (A) 2
   - (B) 3
   - (C) 4
   - (D) -1
   - (E) 0

2. What is the instantaneous rate of change at $x = 3$ of the function f given by $f(x) = \frac{x^2-2}{x+1}$?

(A) $-\frac{17}{16}$

(B) $-\frac{1}{8}$

(C) $\frac{1}{8}$

(D) $\frac{13}{16}$

(E) $\frac{17}{16}$

3. If $x^3 + 2x^2y - 4y = 7$, then when $x = 1, \frac{dy}{dx} =$

(A) $-\frac{9}{2}$

(B) 0

(C) -8

(D) -3

(E) $\frac{7}{2}$

4. If $f''(x) = (x-1)(x+2)^3(x-4)^2$, then the graph of f has inflection points when $x =$

(A) -2 only

(B) 1 only

(C) 1 and 4 only

(D) -2 and 1 only

(E) $-2, 1,$ and 4 only

5. What are all values of k for which $\int_{-2}^{k} x^5 dx = 0$?

(A) -2

(B) 0

(C) 2

(D) -2 and 2

(E) $-2, 0,$ and 2

6. The function f is given by $f(x) = -x^6 + x^3 - 2$. On which of the following intervals is f decreasing?

(A) $(-\infty, 0)$

(B) $\left(-\infty, -\sqrt[3]{\frac{1}{2}}\right)$

(C) $\left(0, \sqrt[3]{\frac{1}{2}}\right)$

(D) $(0, \infty)$

(E) $\left(\sqrt[3]{\frac{1}{2}}, \infty\right)$

7. If $G(x)$ is an antiderivative for $f(x)$ and $G(3) = 6$, then $G(5) =$

(A) $f'(5)$

(B) $6 + f'(5)$

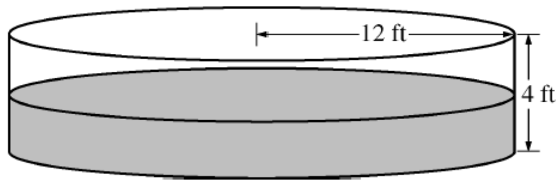
(C) $\int_3^5 f(t)dt$

(D) $\int_3^5 (6 + f(t))dt$

(E) $6 + \int_3^5 f(t)dt$

FREE RESPONSE  **CALCULATOR ACTIVE**  **Your score:___ out of 9**

t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



1. The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time $t = 0$. During the time interval $0 \le t \le 12$ hours, water is pumped into the pool at the rate of $P(t)$ cubic feet per hour. The table above gives values of $P(t)$ for selected values of t. During the same time interval, water is leaking from the pool at the rate $R(t)$ cubic feet per hour, where $R(t) = 25e^{-0.05t}$. (Note: The volume V of a cylinder with radius r and height h is given by $V = \pi r^2 h$.)

(a) Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval $0 \le t \le 12$ hours. Show the computations that lead to your answer.

(b) Calculate the total amount of water that leaked out of the pool during the time interval $0 \le t \le 12$ hours.

(c) Use the results from parts (a) and (b) to approximate the volume of water in the pool at time $t = 12$ hours. Round your answer to the nearest cubic foot.

(d) Find the rate at which the volume of water in the pool is increasing at time $t = 8$ hours. How fast is the water level in the pool rising at $t = 8$ hours? Indicate units of measure in both answers.