Find the antiderivatives of the following.

1. $f'(x) = 4\sqrt[3]{x^2} - \frac{5}{\sqrt[5]{x^3}} + 2$

$f'(x) = 4x^{2/3} - 5x^{-3/5} + 2$

$f(x) = 4 \cdot \frac{3}{5}x^{5/3} - 5 \cdot \frac{5}{2}x^{2/5} + 2x + c$

$f(x) = \frac{12}{5}\sqrt[3]{x^5} - \frac{25}{2}\sqrt[5]{x^2} + 2x + c$

2. $\frac{dy}{dx} = x^{-2} - x^{-1}$

$y = -\frac{1}{x} - \ln x + c$

3. $y' = \sin x + x^{\frac{3}{2}}$

$y = -\cos x + \frac{2}{5}x^{5/2} + c$

$y = -\cos x + \frac{2}{5}\sqrt{x^5} + c$

Evaluate the indefinite integrals.

4. $\int (3x + e^x)\,dx$

$\frac{3}{2}x^2 + e^x + c$

5. $\int \left(3e^x + \frac{9}{x}\right) dx$

$\int 3e^x + 9x^{-1}$

$3e^x + 9\ln x + C$

6. $\int (4 - \cos x)\,dx$

$4x - \sin x + c$

Evaluate the definite integrals using the Fundamental Theorem of Calculus.

7.
$$\int_{\pi}^{\frac{3\pi}{2}} (2 + \cos x)dx = 2x + \sin x \Big]_{\pi}^{3\pi/2}$$

$\left[2\left(\frac{3\pi}{2}\right) + \sin\left(\frac{3\pi}{2}\right)\right] - \left[2(\pi) + \sin\pi\right]$

$(3\pi - 1) - (2\pi + 0)$

$\pi - 1$

8.
$$\int_0^1 (e^x - x)dx = e - \frac{3}{2}$$

Find the function that satisfies the given conditions.

9. $h'(t) = 8t^3 + 5$ and $h(1) = -4$

$h(t) = 2t^4 + 5t + c$

$-4 = 2(1)^4 + 5(1) + c$

$-4 = 2 + 5 + c$

$-4 = 7 + c$

$-11 = c$

$h(t) = 2t^4 + 5t - 11$

10. $\frac{dy}{dx} = 2x + \sin x$ and $y(0) = 4$

$y = x^2 - \cos x + 5$

Find the function that satisfies the given conditions.

11. $f''(x) = x^{-3/2}$ and $f'(4) = 2$ and $f(0) = 0$

$f'(x) = -2x^{-1/2} + c$

$2 = -\frac{2}{\sqrt{4}} + c$

$2 = -1 + c$

$3 = c$

$f'(x) = -2x^{-1/2} + 3$

$f(x) = -4x^{1/2} + 3x + c$

$0 = -4\sqrt{0} + 3(0) + c$

$0 = c$

$f(x) = -4\sqrt{x} + 3x$

12. $f''(x) = \sin x$ and $f'(0) = 1$ and $f(0) = 6$

$f(x) = -\sin x + 2x + 6$

Word Problems!

13. A particle moves along the y-axis with an acceleration of $a(t) = 2$ where t is time in seconds. The particle's velocity at $t = 2$ is 5 cm/sec. The position of the function at $t = 2$ is 10cm. What is the position of the particle at $t = 6$?

$a(t) = 2$

$v(t) = 2t + c$

$5 = 2(2) + c$

$5 = 4 + c$

$1 = c$

$v(t) = 2t + 1$

$x(t) = t^2 + t + c$

$10 = (2)^2 + 2 + c$

$10 = 6 + c$

$4 = c$

$x(t) = t^2 + t + 4$

$x(6) = 6^2 + 6 + 4$

$x(6) = 46$ cm

14. A ball is thrown straight up with a velocity of $v(t) = -32t - 8$ where t is time in seconds and v is ft/sec. The ball is 20 feet in the air at $t = 1$. What is at the initial height of the ball?

44 feet

15. A particle moves along the y-axis with an acceleration of $a(t) = 12t - 4$ with initial velocity of -10 and initial position 0. Find the position of the function at the particle's minimum velocity.

$a(t) = 12t - 4$

$v(t) = 6t^2 - 4t + c$

$v(t) = 6t^2 - 4t - 10$

$x(t) = 2t^3 - 2t^2 - 10t + c$

$x(t) = 2t^3 - 2t^2 - 10t$

Min velocity $a(t) = 0$

$12t - 4 = 0$

$+4 \quad +4$

$\frac{12t}{12} = \frac{4}{12}$

$t = \frac{1}{3}$

x	0	$\frac{1}{3}$	1
$a(t)$	$-$	0	$+$

minimum

$a(t) < 0$ decreasing $(-\infty, \frac{1}{3})$

$a(t) > 0$ increasing $(\frac{1}{3}, \infty)$

$x\left(\frac{1}{3}\right) = 2\left(\frac{1}{3}\right)^3 - 2\left(\frac{1}{3}\right)^2 - 10\left(\frac{1}{3}\right)$

$x\left(\frac{1}{3}\right) = 2\left(\frac{1}{27}\right) - 2\left(\frac{1}{9}\right) - 10\left(\frac{1}{3}\right)$

$x\left(\frac{1}{3}\right) = \frac{2}{27} - \frac{2}{9} - \frac{10}{3}$

$x\left(\frac{1}{3}\right) = \frac{2}{27} - \frac{6}{27} - \frac{90}{27}$

$x\left(\frac{1}{3}\right) = -\frac{94}{27}$

16. The graph of f includes the point (2,6) and the slope of the tangent line to f at any point x is given by the expression $3x + 4$. Find $f(-2)$.

$f(-2) = -10$

17. A coin is dropped from a 850 foot building The velocity of the coin is $v(t) = -16t$. Find the both the position function and acceleration function.

$v(t) = -16t$

$s(t) = -8t^2 + 850$

$a(t) = v'(t) = -16$

18. A particle moves along the x-axis with a velocity of $(t) = \sqrt[5]{x^2} - \frac{1}{x^2}$. At $t = 1$ second the position of the particle is 3 inches. What is the particle's position at $t = 8$?

$\frac{829}{40}$ inches

19. A particle moves along the x-axis with a velocity of $v(t) = 1 - \sin t$. At $t = \pi$ seconds the position of the particle is π inches. What is the position of the particle at $t = \frac{3\pi}{2}$?

$v(t) = 1 - \sin t$

$x(t) = t + \cos t + C$

$\pi = \pi + \cos(\pi) + C$

$\pi = \pi - 1 + C$

$1 = C \longrightarrow x(t) = t + \cos t + 1$

$x\left(\frac{3\pi}{2}\right) = \frac{3\pi}{2} + \cos\left(\frac{3\pi}{2}\right) + 1$

$x\left(\frac{3\pi}{2}\right) = \frac{3\pi}{2} + 0 + 1$

$x\left(\frac{3\pi}{2}\right) = \frac{3\pi}{2} + 1$

20. A particle moves along the y-axis with a velocity of $v(t) = \frac{1}{t} - \frac{t^2}{3} + 2$. At $t = 1$ seconds the position of the particle is 8 meters. Find the both the acceleration and position function.

$a(t) = -\frac{1}{t^2} - \frac{2}{3}t$

$x(t) = \ln t - \frac{1}{9}t^3 + 2t + \frac{55}{9}$

8.3 Antiderivatives

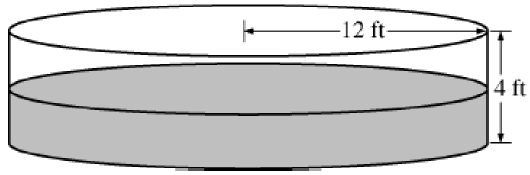
TEST PREP

MULTIPLE CHOICE

1. B
2. E
3. A
4. D
5. D
6. E
7. E

FREE RESPONSE **CALCULATOR ACTIVE** **Your score:___ out of 9**

t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time $t = 0$. During the time interval $0 \le t \le 12$ hours, water is pumped into the pool at the rate $P(t)$ cubic feet per hour. The table above gives values of $P(t)$ for selected values of t. During the same time interval, water is leaking from the pool at the rate $R(t)$ cubic feet per hour, where $R(t) = 25e^{-0.05t}$. (Note: The volume V of a cylinder with radius r and height h is given by $V = \pi r^2 h$.)

(a) Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval $0 \le t \le 12$ hours. Show the computations that lead to your answer.

(b) Calculate the total amount of water that leaked out of the pool during the time interval $0 \le t \le 12$ hours.

(c) Use the results from parts (a) and (b) to approximate the volume of water in the pool at time $t = 12$ hours. Round your answer to the nearest cubic foot.

(d) Find the rate at which the volume of water in the pool is increasing at time $t = 8$ hours. How fast is the water level in the pool rising at $t = 8$ hours? Indicate units of measure in both answers.

(a) $\int_0^{12} P(t)\,dt \approx 46 \cdot 4 + 57 \cdot 4 + 62 \cdot 4 = 660 \text{ ft}^3$

$2: \begin{cases} 1: \text{midpoint sum} \\ 1: \text{answer} \end{cases}$

(b) $\int_0^{12} R(t)\,dt = 225.594 \text{ ft}^3$

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$

(c) $1000 + \int_0^{12} P(t)\,dt - \int_0^{12} R(t)\,dt = 1434.406$

At time $t = 12$ hours, the volume of water in the pool is approximately 1434 ft^3.

$1: \text{answer}$

(d) $V'(t) = P(t) - R(t)$

$V'(8) = P(8) - R(8) = 60 - 25e^{-0.4} = 43.241 \text{ or } 43.242 \text{ ft}^3/\text{hr}$

$V = \pi(12)^2 h$

$\frac{dV}{dt} = 144\pi \frac{dh}{dt}$

$\left.\frac{dh}{dt}\right|_{t=8} = \frac{1}{144\pi} \cdot \left.\frac{dV}{dt}\right|_{t=8} = 0.095 \text{ or } 0.096 \text{ ft/hr}$

$4: \begin{cases} 1: V'(8) \\ 1: \text{equation relating } \frac{dV}{dt} \text{ and } \frac{dh}{dt} \\ 1: \left.\frac{dh}{dt}\right|_{t=8} \\ 1: \text{units of ft}^3/\text{hr and ft/hr} \end{cases}$