

9.1 The 2nd Fundamental Theorem of Calculus

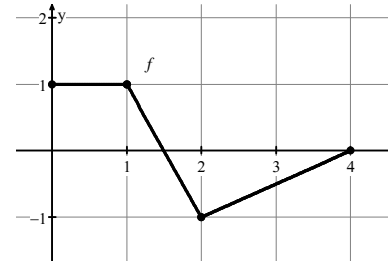
Name: _____

Write your questions
and thoughts here!**Notes**

Function Defined by an Integral:

$$F(x) = \int_0^x f(t) dt$$

1. Use the function f in the figure and the function F defined by $F(x) = \int_0^x f(t) dt$ on the interval $0 \leq x \leq 4$.



- a) Complete the table.

x	0	1	2	3	4
$F(x)$					

- b) When does $F(x)$ have a minimum? c) When does $F(x)$ have a maximum?

- d) Integrate $f(t)$.

- e) Now take the derivative.

Second Fundamental Theorem of Calculus

If $F(x) = \int_a^x f(t) dt$, where a is constant and f is a continuous function, then

If $F(x) = \int_a^{g(x)} f(t) dt$, where a is constant, f is a continuous function, and g is a differentiable function, then

9.1 The 2nd FTC

Notes

Write your questions
and thoughts here!

Find $F'(x)$.

1. $F(x) = \int_2^x (3t^2 + 4t) dt$

2. $F(x) = \int_{\pi/2}^{x^3} \sin(t) dt$

3. $F(x) = \int_1^{4x} h(t) dt$

4. $F(x) = \int_{-x}^x 5t dt$

5. $F(x) = \int_{2x}^{3x} (t^2 - t) dt$



Now
summarize
what you
learned!

9.1 The 2nd FTC

Calculus

Name: _____

Practice

Find $F'(x)$.

1. $F(x) = \int_2^x t^3 dt$

2. $F(x) = \int_0^x 5 dt$

3. $F(x) = \int_{-1}^x (4t - t^2) dt$

4. $F(x) = \int_{\pi}^x \cos(t) dt$

5. $F(x) = \int_1^{x^2} t^3 dt$

6. $F(x) = \int_{\pi}^{x^2} \sin(t) dt$

Find $F'(x)$.

7. $F(x) = \int_{\pi}^{\sin x} \frac{1}{t} dt$

8. $F(x) = \int_4^{x^2} 3\sqrt{t} dt$

9. $F(x) = \int_0^{3x} 2t dt$

10. $F(x) = \int_0^{\tan x} t^2 dt$

11. $F(x) = \int_3^{x^2} \tan(t) dt$

12. $F(x) = \int_3^{g(x)} \sec(t) dt$

13. $F(x) = \int_1^{2x} f(t) dt$

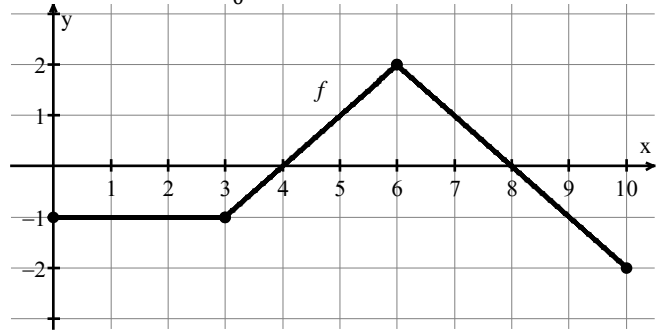
14. $F(x) = \int_x^{x+2} (4t + 1) dt$

15. $F(x) = \int_{-x^2}^x (3t - 1) dt$

16. $F(x) = \int_{-x}^x t^3 dt$

17. $F(x) = \int_{2x}^{3x} t^2 dt$

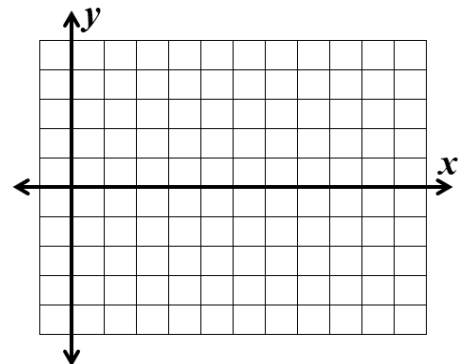
18. Use the function f in the figure and the function g defined by $g(x) = \int_0^x f(t) dt$.



a) Complete the table

x	0	1	2	3	4	5	6	7	8	9	10
$g(x)$											

b) Plot the points from the table in part (a).



From #18 on the previous page.

c) Where does g have its minimum? Explain.

d) Which four consecutive points are collinear? Explain.

e) Between which two consecutive points does g increase at the greatest rate? Explain.

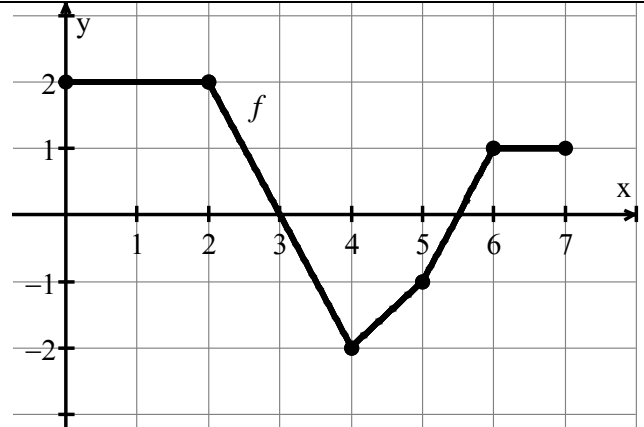
19. Use the function f in the figure and the function g defined by $g(x) = \int_0^x f(t) dt$ to answer the following questions.

a) Find $g(4)$.

b) At what x -values does g have a minimum?

c) At what x -values does g have a maximum?

d) Let h be the function defined by $h(x) = \frac{f(x)}{x^2+1}$. Find $h'(3)$.



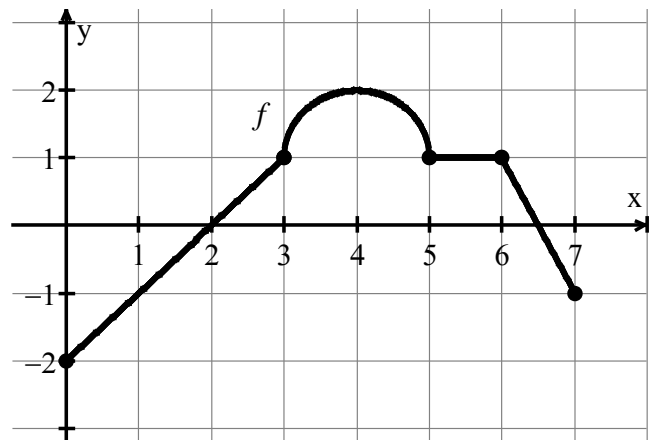
20. Use the function f in the figure and the function h defined by $h(x) = \int_0^x f(t) dt$ to answer the following questions.

a) Find $h(5)$.

b) At what x -values does h have a minimum?

c) At what x -values does h have a maximum?

d) Let g be the function defined by $g(x) = f(x)(x^2 - 3)$. Find $g'(1)$.



21. $H(x) = \int_{\pi/2}^x t \cos(t) dt$ for $0 < x < 2\pi$.

a) Determine the critical numbers of $H(x)$.

b) Determine which critical number corresponds to a relative maximum value of $H(x)$. Justify your answer.

c) Determine which critical number corresponds to a relative minimum value of $H(x)$. Justify your answer.

9.1 The 2nd Fundamental Theorem of Calculus

Test Prep

1. Let g be the function given by: $g(x) = \int_0^x \sin t^2 dt$ for $-1 \leq x \leq 3$. On which of the following intervals is g decreasing?



(A) $-1 \leq x \leq 0$

(B) $0 \leq x \leq 1.772$

(C) $1.253 \leq x \leq 2.171$

(D) $1.772 \leq x \leq 2.507$

(E) $2.802 \leq x \leq 3$

2. $\frac{d}{dt} \int_2^{t^4} e^{x^2} dx =$

(A) $e^{t^8} - e^4$

(B) $4t^3 e^{t^8} - e^4$

(C) e^{t^8}

(D) $4t^3 e^{t^8}$

(E) Cannot be determined because $\int e^{x^2}$ cannot be determined

3. If $f(x) = x\sqrt{3x-4}$, then $f'(x) =$

(A) $\frac{3x-1}{\sqrt{3x-4}}$

(B) $\frac{6x-7}{\sqrt{3x-4}}$

(C) $\frac{9x-8}{2\sqrt{3x-4}}$

(D) $\frac{6x-5}{\sqrt{3x-4}}$

(E) $\frac{6x-5}{2\sqrt{3x-4}}$

4. Find $\lim_{h \rightarrow 0} \frac{(x+h)^9 \sin(x+h)^2 - x^9 \sin x^2}{h}$

(A) $x^8(2x \sin x \cos x + 9 \sin x^2)$

(B) 0

(C) $x^8(2x^2 \cos x^2 + 9 \sin x^2)$

(D) ∞

(E) The limit does not exist.

5. Boats A and B leave the same place at the same time. Boat A heads due north at 12 km/hr. Boat B heads due east at 18 km/hr. After 2.5 hours, how fast is the distance between the boats increasing (in km/hr)?



(A) 21.63

(B) 31.20

(C) 75.00

(D) 9.84

(E) 54.08

6. Let f be the function given by $f(x) = 3^x$. For what value of x is the slope of the line tangent to the curve at $(x, f(x))$ equal to 1?



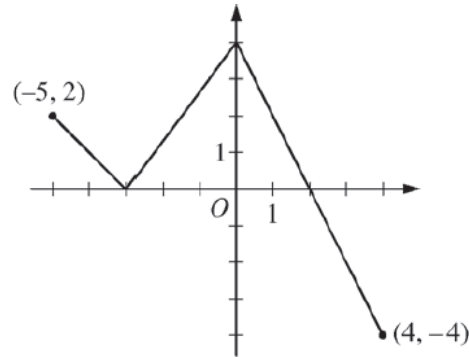
(A) 1.099

(B) 0.086

(C) 0

(D) -0.086

(E) -1.099



Graph of f

The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above. Let g be defined by $g(x) = \int_{-3}^x f(t) dt$.

- Find $g(3)$.
- On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.
- The function h is defined by $h(x) = \frac{g(x)}{5x}$. Find $h'(3)$.
- The function p is defined by $p(x) = f(x^2 - x)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.