9.1 The 2nd FTC

Calculus

Name: Solutions

Practice

Find $F'(x)$.

1. $F(x) = \int_2^x t^3\,dt$ $\boxed{F'(x) = x^3}$	2. $F(x) = \int_0^x 5\,dt$ $\boxed{F'(x) = 5}$	3. $F(x) = \int_{-1}^x (4t - t^2)\,dt$ $\boxed{F'(x) = 4x - x^2}$
4. $F(x) = \int_\pi^x \cos(t)\,dt$ $\boxed{F'(x) = \cos x}$	5. $F(x) = \int_1^{x^2} t^3\,dt$ $F'(x) = (x^2)^3 \cdot 2x$ $\boxed{F'(x) = 2x^7}$	6. $F(x) = \int_\pi^{x^2} \sin(t)\,dt$ $\boxed{F'(x) = 2x\sin(x^2)}$

Find $F'(x)$.

7. $F(x) = \int_{\pi}^{\sin x} \frac{1}{t}\,dt$

$F'(x) = \frac{1}{\sin x} \cdot \cos x$

$\boxed{F'(x) = \cot x}$

8. $F(x) = \int_{4}^{x^2} 3\sqrt{t}\,dt$

$\boxed{F'(x) = 6x^2}$

9. $F(x) = \int_{0}^{3x} 2t\,dt$

$F'(x) = 2(3x) \cdot 3$

$\boxed{F'(x) = 18x}$

10. $F(x) = \int_{0}^{\tan x} t^2\,dt$

$\boxed{F'(x) = \tan^2 x \sec^2 x}$

11. $F(x) = \int_{3}^{x^2} \tan(t)\,dt$

$F'(x) = \tan(x^2) \cdot 2x$

$\boxed{F'(x) = 2x\tan(x^2)}$

12. $F(x) = \int_{3}^{g(x)} \sec(t)\,dt$

$\boxed{F'(x) = g'(x)\sec(g(x))}$

13. $F(x) = \int_{1}^{2x} f(t)\,dt$

$\boxed{F'(x) = 2f(2x)}$

14. $F(x) = \int_{x}^{x+2} (4t+1)\,dt$

$\boxed{F'(x) = 8}$

15. $F(x) = \int_{-x^2}^{x} (3t-1)\,dt$

$F(x) = \int_{-x^2}^{0} (3t-1)\,dt + \int_{0}^{x} (3t-1)\,dt$

$F(x) = -\int_{0}^{x^2} (3t-1) + \int_{0}^{x} (3t-1)$

$F'(x) = -(3(x^2)-1) \cdot (-2x) + (3x-1)$

$\boxed{F'(x) = -6x^3 + x - 1}$

16. $F(x) = \int_{-x}^{x} t^3\,dt$

$\boxed{F'(x) = 0}$

17. $F(x) = \int_{2x}^{3x} t^2\,dt$

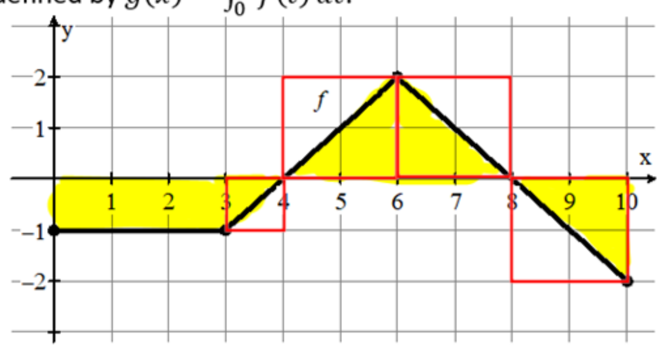
$F(x) = \int_{2x}^{0} t^2 + \int_{0}^{3x} t^2$

$F(x) = -\int_{0}^{2x} t^2 + \int_{0}^{3x} t^2$

$F'(x) = -(2x)^2(2) + (3x)^2(3)$

$\boxed{F'(x) = 19x^2}$

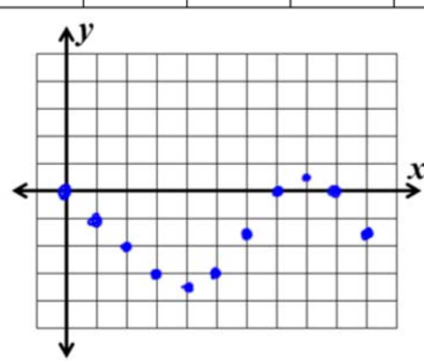
18. Use the function f in the figure and the function g defined by $g(x) = \int_0^x f(t)\,dt$.



a) Complete the table

x	0	1	2	3	4	5	6	7	8	9	10
$g(x)$	0	−1	−2	−3	−3.5	−3	−1.5	0	0.5	0	−1.5

b) Plot the points from the table in part (a).



From #18 on the previous page.

c) Where does g have its minimum? Explain.

x=4 because g'(x) [which is the same as f(x)] changes from negative to positive.

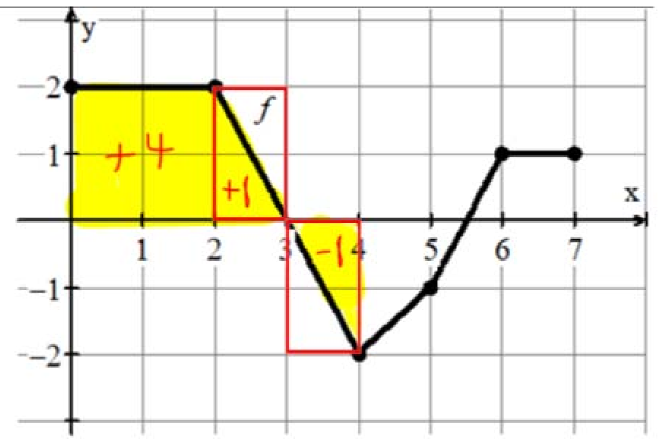
d) Which four consecutive points are collinear? Explain.

x=0, 1, 2, 3, because g'(x) is constant, giving a linear relationship.

e) Between which two consecutive points does g increase at the greatest rate? Explain.

Between x=6 and x=7 because this is where there is the most positive area under the curve.

19. Use the function f in the figure and the function g defined by $g(x) = \int_0^x f(t)\,dt$ to answer the following questions.



a) Find $g(4)$.

$4+1-1=\boxed{4}$

b) At what x-values does g have a minimum?

$\boxed{x=0,\ 5.5}$

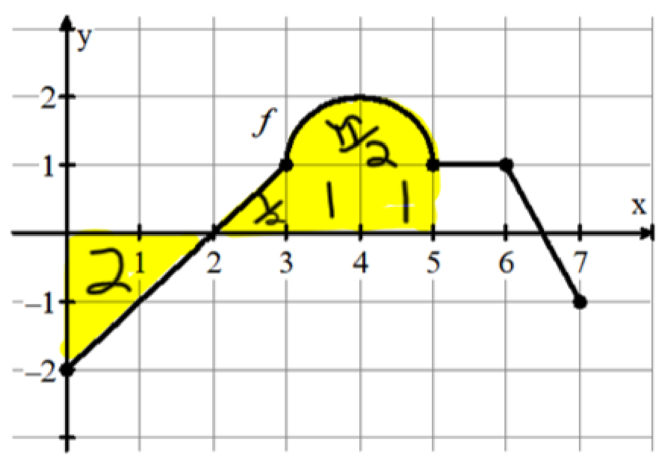
c) At what x-values does g have a maximum?

$\boxed{x=3,\ 7}$

d) Let h be the function defined by $h(x) = \frac{f(x)}{x^2+1}$. Find $h'(3)$.

$$\frac{f'(x)(x^2+1) - f(x)(2x)}{(x^2+1)^2} \longrightarrow \frac{(-2)(10)-(0)(6)}{100} = \boxed{-\frac{1}{5}}$$

20. Use the function f in the figure and the function h defined by $h(x) = \int_0^x f(t)\,dt$ to answer the following questions.



a) Find $h(5)$.

$-2+\frac{1}{2}+2+\frac{\pi}{2} = \boxed{\frac{\pi}{2}+\frac{1}{2}}$

b) At what x-values does h have a minimum?

$\boxed{x=2,\ 7}$

c) At what x-values does h have a maximum?

$\boxed{x=0,\ 6.5}$

d) Let g be the function defined by $g(x) = f(x)(x^2-3)$. Find $g'(1)$.

$g'(x) = f'(x)(x^2-3) + f(x)(2x)$

$f'(1)(-2) + f(1)(2) = (1)(-2) + (-1)(2) = \boxed{-4}$

21. $H(x) = \int_{\pi/2}^{x} t\cos(t)\,dt$ for $0 < x < 2\pi$.

a) Determine the critical numbers of $H(x)$.

$H' = 0$ or H' DNE $\quad \boxed{x = 0}$ or $\cos x = 0$

$x\cos(x) = 0 \quad \boxed{x = \pi/2}$ or $\boxed{x = 3\pi/2}$

b) Determine which critical number corresponds to a relative maximum value of $H(x)$. Justify your answer.

$H''(x) = \cos(x) - x\sin(x)$

$H''(0) = 1$

$H''(\pi/2) = 0 - \pi/2(1) \quad H''(3\pi/2) = 0 - (3\pi/2)(-1)$

$\boxed{x = \pi/2 \text{ is a max b/c } H'(\pi/2) = 0 \text{ and } H''(\pi/2) < 0.}$

c) Determine which critical number corresponds to a relative minimum value of $H(x)$. Justify your answer.

$\boxed{x = 0 \text{ and } x = 3\pi/2 \text{ are minimums b/c } H' = 0 \text{ and } H'' > 0.}$

Test Prep: 1D, 2D, 3C, 4C, 5A, 6D

Free Response Scoring Guide

Use this only AFTER you have attempted the problem on your own.

Solutions	Points
(a) $g(3) = \int_{-3}^{3} f(t)\,dt = 6 + 4 - 1 = 9$	1 : answer
(b) $g'(x) = f(x)$ The graph of g is increasing and concave down on the intervals $-5 < x < -3$ and $0 < x < 2$ because $g' = f$ is positive and decreasing on these intervals.	$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$
(c) $h'(x) = \dfrac{5xg'(x) - g(x)5}{(5x)^2} = \dfrac{5xg'(x) - 5g(x)}{25x^2}$ $h'(3) = \dfrac{(5)(3)g'(3) - 5g(3)}{25 \cdot 3^2}$ $= \dfrac{15(-2) - 5(9)}{225} = \dfrac{-75}{225} = -\dfrac{1}{3}$	$3 : \begin{cases} 2 : h'(x) \\ 1 : \text{answer} \end{cases}$
(d) $p'(x) = f'(x^2 - x)(2x - 1)$ $p'(-1) = f'(2)(-3) = (-2)(-3) = 6$	$3 : \begin{cases} 2 : p'(x) \\ 1 : \text{answer} \end{cases}$