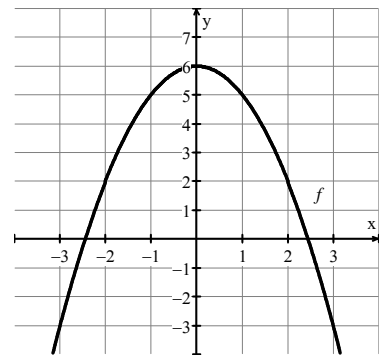


Write your questions
and thoughts here!**Notes****Recall:****Average Rate of Change:****Mean Value Theorem (MVT) for Derivatives:****Average Value of a Function:**

1. Find the average value of $f(x) = 6 - x^2$ on $[-1, 3]$.



When does the function assume this value?

MVT for Integrals:

Comparing **average rate of change** (secant slope) and **average value** of a function.

Set up the equation for each question, but do not solve it. What units will the ANSWER be?

2. $h(t) = -16t^2 + 41t + 10$. h is height (feet) and t is time (seconds).
 - a. What is the average height during the first 3 seconds?
 - b. What is the average velocity during the first 3 seconds?
3. $r(x) = 2 \sin x - 1$, where r is the rate at which Mr. Brust's waistline is changing (inches per month) and x is time (months).
 - a. What is the average rate that Mr. Brust's waistline changes from the 10th to the 12th month?
 - b. What is the average change of this rate during the first 5 months?

9.3 Average Value of a Function

Calculus

Name: _____

Practice

Find the average value of each function on the given interval.

1. $f(x) = x^2$ on $[2, 4]$

2. $f(x) = x^2 - 2x$ on $[0, 3]$

3. $f(x) = \sin x$ on $[0, \pi]$

4. $f(x) = \sqrt{x}$ on $[0, 16]$

5. $f(x) = \frac{1}{x^2}$ on $[-4, -2]$

6. $f(x) = 2e^x$ on $[-3, 1]$

On the given interval, find the x -value where the function is equivalent to the average value on that interval.

7. $f(x) = 2x - 2$ on $[1, 4]$

8. $f(x) = -\frac{x^2}{2}$ on $[0, 3]$

9. $f(x) = 2x^2 + 16x + 28$ on $[-5, -2]$

Find the average rate of change on the given interval.

10. $f(x) = -(2x - 6)^{\frac{2}{3}}$ on $[1, 3]$

11. $y = x^3 - 2x^2 + 2$ on $[-1, 1]$

12. $y = \ln \sqrt{x}$ on $[1, e]$

For 13-14, find where the instantaneous rate of change is equivalent to the average rate of change.

13. $y = x^2 - 4x + 3$ on $[0, 4]$

14. $y = \sqrt{9 - 3x}$ on $[-2, 3]$

15. The temperature (in °F) t hours after 9 AM is approximated by the function $T(t) = 50 + 14 \sin \frac{\pi t}{12}$. Find the average temperature during the time period 9 AM to 9 PM.



16. The depth of water in Mr. Brust's hot tub can be represented by the formula $h(t) = -\cos(t) + 2$, where t is the time in minutes since he begins pouring in water and $h(t)$ is measured in feet. What is the average depth of the water during the first three minutes? Set up the expression and use a calculator to help solve.



17. Find the number(s) b such that the average value of $y = 2 + 7x - x^3$ on the interval $[0, b]$ is equal to 2.

18. Find the number(s) b such that the average value of $y = 2 + 6x - 3x^2$ on the interval $[0, b]$ is equal 3. *Hint: quadratic formula needed!*

19. 2004 A Q1 c-d

Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4 \sin\left(\frac{t}{2}\right) \text{ for } 0 \leq t \leq 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

(c) What is the average value of the traffic flow over the time interval $10 \leq t \leq 15$? Indicate units of measure.



(d) What is the average rate of change of the traffic flow over the time interval $10 \leq t \leq 15$? Indicate units of measure.

20. 2005 A Q3 b-d



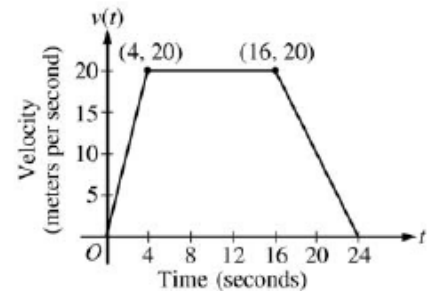
Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ ($^{\circ}\text{C}$)	100	93	70	62	55

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius ($^{\circ}\text{C}$), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

- (b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.
- (c) Find $\int_0^8 T'(x) dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x) dx$ in terms of the temperature of the wire.
- (d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

21. 2005 A Q5 d

A car is traveling on a straight road. For $0 \leq t \leq 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph to the right.



- (d) Find the average rate of change of v over the interval $8 \leq t \leq 20$. Does the Mean Value Theorem guarantee a value of c , for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

9.3 Average Value of a Function

Test Prep

1. The average value of $f(x) = x^3$ over the interval $a \leq x \leq b$ is

(A) $3b + 3a$

(B) $b^2 + ab + a^2$

(C) $\frac{b^3+a^3}{2}$

(D) $\frac{b^3-a^3}{2}$

(E) $\frac{b^4-a^4}{4(b-a)}$

2. The average value of the function $f(x) = \ln^2 x$ on the interval $[2, 4]$ is



- (A) -1.204 (B) 1.204 (C) 2.159 (D) 2.408 (E) 8.636
-

3. The function f is continuous on the closed interval $[1, 3]$ and has the values given in the table. The equation $g(x) = \frac{5}{4}$ must have at least two intersections with $f(x)$ in the interval $[1, 3]$ if $k =$

x	1	2	3
$f(x)$	2	k	4

- (A) $\frac{1}{4}$ (B) $\frac{3}{2}$ (C) 2 (D) $\frac{9}{4}$ (E) 3
-

4. A particle moves along the x -axis so that its position at time t is given by $x(t) = t^2 - 7t + 12$. For what value of t is the velocity of the particle zero?

- (A) 2.5 (B) 3 (C) 3.5 (D) 4 (E) 4.5
-

5. The function g is given by $g(x) = \frac{3x^2}{e^{3x}}$. On which of the following intervals is g increasing?

- (A) $(-\infty, 0)$ (B) $(-\infty, \frac{2}{3})$ (C) $(0, \frac{2}{3})$ (D) $(0, \infty)$ (E) $(\frac{2}{3}, \infty)$