Calculus

Write your questions and thoughts here!

9.4 Net Change

Name:________________________

Notes

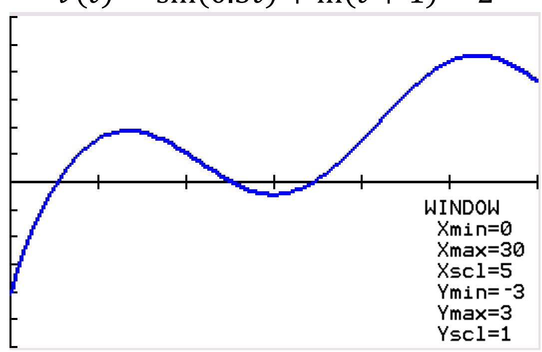
When you integRATE a RATE, you get net change!

$$\int \qquad\qquad = $$

Example 1: Mr. Brust is driving across town to Mr. Sullivan's house to play with a new set of Star Wars figures. Mr. Brust's speed would obviously vary throughout the drive, but because he is so cool, he came up with a function that represents his velocity (miles per minute) at any given time t (minutes) since he left his house during the 30 minute drive.

Set up the expressions for the following scenarios. Use a calculator to solve.

$v(t) = \sin(0.3t) + \ln(t+1) - 2$



a. How far is Mr. Brust from his house after 10 minutes?

b. How far is Mr. Brust from his house after 15 minutes?

c. If Mr. Brust arrives at Mr. Sullivan's house after 30 minutes, how far away does he live?

d. How many miles did Mr. Brust drive?

Write your questions and thoughts here!

$\int \qquad =$ $\qquad\qquad \int \qquad =$

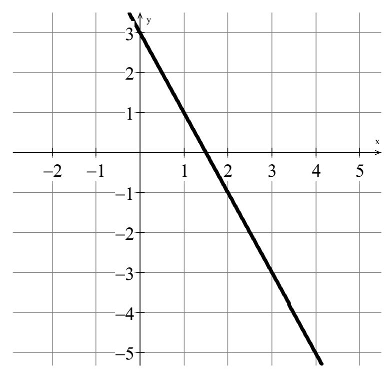
Don't get this confused with: $|\text{velocity}| = \text{speed}$

Example 2: A particle's velocity is given by $v(t) = t^3 - 2t^2 + 1$. If $x(t)$ represents the position of the particle along the x-axis, find the following:

a. The position of the particle after 3 seconds if $x(0) = 5$.

b. The position of the particle after 2 seconds if $x(1) = -2$.

Example 3: What is the total distance traveled by a particle during the first 4 seconds if the particle's velocity function is given by $v(t) = -2t + 3$? Show the set up AND your work.



Example 4: If $H(-\pi) = 12$ and $H'(t) = \cos(t)$, what is $H\left(\frac{3\pi}{2}\right)$?

Now summarize what you learned!

9.4 Net Change

Calculus

Practice

1. Mr. Kelly leaves for a trip at 3:00 p.m. (time $t = 0$) and drives with velocity $v(t) = 60 - \frac{1}{2}t$ miles per hour, where t is measured in hours.

   a. Find $\int_0^2 v(t)\,dt$

   b. Explain the meaning of your answer to part *a* in the context of this problem.

2. Mr. Brust's placenta tree grows in height at a rate of $r(t) = \sqrt[4]{t}$ feet per year where t is measured in years.

   a. Find $\int_1^{16} r(t)\,dt$

   b. Explain the meaning of your answer to part *a* in the context of this problem.

3. A ball is thrown at the ground from the top of a tall building. The speed of the ball in meters per second is $v(t) = 9.8t + v_0$, where t denotes the number of seconds since the ball has been thrown and v_0 is the initial speed of the ball (also in meters per second). If the ball travels 25 meters during the first 2 seconds after it is thrown, what was the initial speed of the ball?

4. Tom Sawyer is painting a fence at a rate of $(200 - 4t)$ square feet per hour, where t is the number of hours since he started painting. If the fence is 800 square feet, how long will it take him to finish painting the fence? Round your answer to the nearest minute.

5. A particle's velocity is given by $v(t) = 2t - 8$, where t is measured in seconds, v is measured in feet per second, and $s(t)$ represents the particle's position.

   (a) If $s(0) = 2$, what is the value of $s(3)$?

   (b) What is the net change in distance over the first 5 seconds?

   (c) What is the total distance traveled by the particle during the first 5 seconds? Show the set up AND your work.

6. A particle's velocity is given by $v(t) = t^2 + 2t - 15$, where t is measured in minutes, v is measured in meters per minute, and $s(t)$ represents the particle's position.
(a) If $s(1) = -3$, what is the value of $s(4)$?

(b) What is the net change in distance over the first 5 minutes?

(c) What is the total distance traveled by the particle during the first 5 minutes? Show the set up AND your work.

---

7. A particle's velocity is given by $v(t) = t^2 - 3t + 2$, where t is measured in hours, v is measured in miles per hour, and $s(t)$ represents the particle's position.
(a) If $s(2) = 5$, what is the value of $s(5)$?

(b) What is the net change in distance over the first 5 hours?

(c) What is the total distance traveled by the particle during the first 5 hours? Show the set up AND your work.

---

8. A particle's velocity is given by $v(t) = \cos t$, where t is measured in months, v is measured in kilometers per month, and $s(t)$ represents the particle's position.
(a) If $s\left(\frac{\pi}{6}\right) = 10$, what is the value of $s\left(\frac{3\pi}{2}\right)$?

(b) What is the net change in distance over the first π months?

(c) What is the total distance traveled by the particle during the first π months? Show the set up AND your work.

9. A particle's velocity is given by $v(t) = 6\cos 3t$, where t is measured in days, v is measured in yards per day, and $s(t)$ represents the particle's position.

(a) If $s(0) = 5$, what is the value of $s\left(\frac{\pi}{2}\right)$?

(b) What is the net change in distance over the first $\frac{\pi}{2}$ days?

(c) What is the total distance traveled by the particle during the first $\frac{\pi}{2}$ days? Show the set up, but use a calculator to find the answer.

---

10. Rate of consumption of oil in the United States during the 1980s (in billions of barrels per year) is modeled by the function

$$R(t) = 27.08e^{\frac{t}{25}}$$

where t is the number of years after January 1, 1980. Find the total consumption of oil in the United States during the 1980s.

---

11. $f'(x) = 6x^2 + 3$ and $f(1) = 4$. What is the value of $f(3)$?

12. $g'(x) = \cos x$ and $g(\pi) = 7$. What is the value of $g\left(\frac{3\pi}{2}\right)$?

13. $h'(x) = 5 - 3x$ and $h(-1) = 0$. What is the value of $h(2)$?

14. $f'(x) = 3x^2 - x$ and $f(2) = 6$. What is the value of $f(3)$?

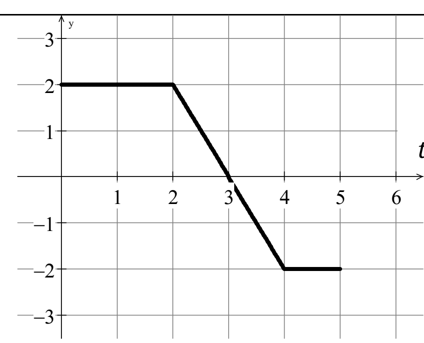
---

15. The graph to the right shows the velocity of an object moving along the x-axis over a 5-second period.

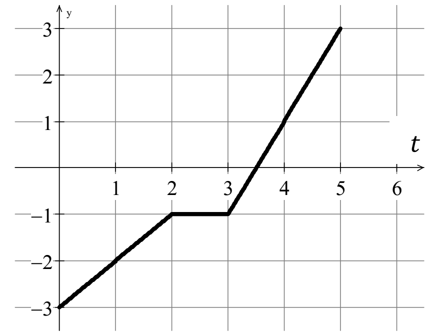
a) If the objected started 2 meters to the right, where is the object after 3 seconds?

b) Where is the object after 5 seconds?

c) Find the total distance traveled by the object over the 5-second period.



16. The graph to the right shows the **velocity** of an object moving along the x-axis over a 5-second period.
    a) Find the total distance traveled by the object over the 5-second period.



    b) At time $t = 2$, the particle is at the point where $x = 4$. Where was the particle at time $t = 0$?

---

17. Fuel is pumped from an underground tank at the rates indicated in the table. The fuel is sold for $1.50/gallon.

    (a) Explain the meaning of $\int_0^{24} P(t)\,dt$. Use correct units.

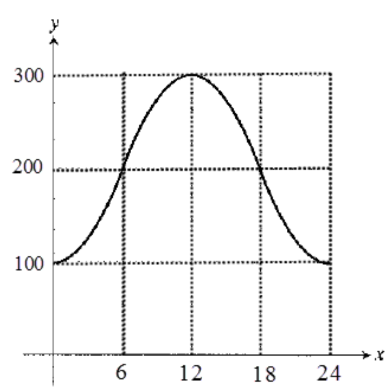
    (b) Use a left-hand Riemann sum with 6 subintervals to approximate the total amount of money earned from the sale of fuel over the 24-hour period.

t (hours)	Pump rate, *P(t)* (gals/hour)
0	250
4	300
8	400
12	500
16	200
20	650
24	450

9.4 Net Change

Test Prep

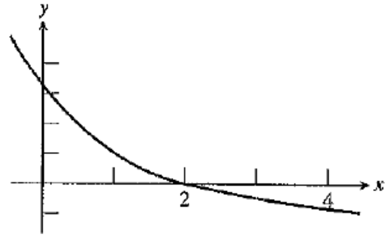
1. The production rate of cola, in thousands of gallons per hour, at the production plant on July 1 is shown in the graph. Of the following, which best approximates the total number of thousands of gallons of cola that were produced that day?



(A) 800 (B) 4200 (C) 4800

(D) 5000 (E) 5400

2. The graph of a twice-differentiable function f is shown. Which of the following is true?

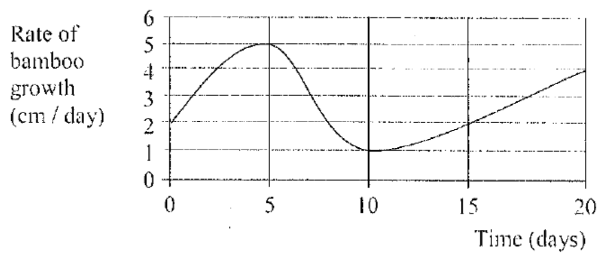


(A) $f(2) < f'(2) < f''(2)$ (B) $f(2) < f''(2) < f'(2)$ (C) $f'(2) < f(2) < f''(2)$

(D) $f''(2) < f(2) < f'(2)$ (E) $f''(2) < f'(2) < f(2)$

---

3. The graph below shows the rate of bamboo growth $g(t)$ in centimeters (cm) per day over a 20-day period.



If the bamboo is 60 cm tall at time $t = 10$ days, approximately how tall is it at $t = 20$ days?

(A) 25 cm (B) 64 cm (C) 70 cm (D) 82 cm (E) 100 cm

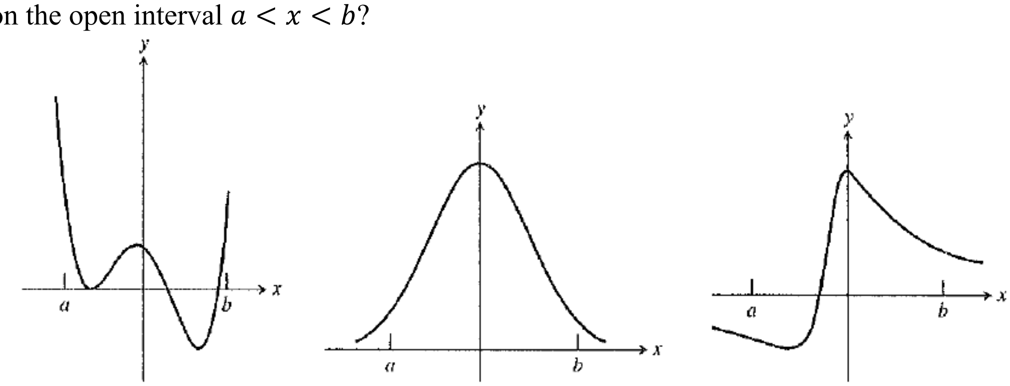
---

4. Find the total distance traveled (to three decimal places) in the first four seconds, for a particle whose velocity is given by $v(t) = 7e^{-t^2}$; where t stands for time.

(A) 0.976 (B) 6.204 (C) 6.359 (D) 12.720 (E) 7.000

---

5. The graphs of the derivatives of the function f, g, and h are shown. Which of the functions have a relative minimum on the open interval $a < x < b$?



$y = f'(x)$ $y = g'(x)$ $y = h'(x)$

(A) f only (B) g only (C) h only (D) f and h only (E) f, g, and h

6. A particle moves in a straight line, and its velocity at any time t is given by $v(t) = 5 - e^t$. What is the total distance the particle travels from $t = 0$ to $t = 3$?

(A) 4.086 (B) 5.086 (C) 11.086 (D) 12.180 (E) 19.086

FREE RESPONSE **Your score: _______ out of 9**

2013 AB1

ACTIVE

On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday ($t = 0$), the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

(c) Is the amount of unprocessed gravel that arrives at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.