9.4 Net Change
Calculus

Solutions

Practice

1. Mr. Kelly leaves for a trip at 3:00 p.m. (time $t = 0$) and drives with velocity $v(t) = 60 - \frac{1}{2}t$ miles per hour, where t is measured in hours.

a. Find $\int_0^2 v(t)\,dt$

$60t - \frac{1}{4}t^2 \Big|_0^2$

$60(2) - \frac{1}{4}(2)^2 - 0$

$120 - 1 = \boxed{119}$

b. Explain the meaning of your answer to part *a* in the context of this problem.

Mr. Kelly traveled 119 miles between 3:00 and 5:00 p.m.

2. Mr. Brust's placenta tree grows in height at a rate of $r(t) = \sqrt[4]{t}$ feet per year where t is measured in years.

a. Find $\int_1^{16} r(t)\,dt$

$\boxed{\frac{124}{5}}$

b. Explain the meaning of your answer to part *a* in the context of this problem.

Mr. Brust's tree grew 24.8 feet between the end of year one and the end of year 16.

3. A ball is thrown at the ground from the top of a tall building. The speed of the ball in meters per second is $v(t) = 9.8t + v_0$, where t denotes the number of seconds since the ball has been thrown and v_0 is the initial speed of the ball (also in meters per second). If the ball travels 25 meters during the first 2 seconds after it is thrown, what was the initial speed of the ball?

$25 = \int_0^2 9.8t + v_0\,dt$

$25 = 4.9t^2 + v_0 t \Big|_0^2$

$25 = 4.9(4) + v_0(2) - 0$

$2.7 = v_0$ $\quad \boxed{2.7 \text{ meters/sec}}$

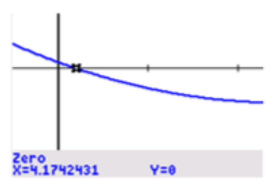
4. Tom Sawyer is painting a fence at a rate of $(200 - 4t)$ square feet per hour, where t is the number of hours since he started painting. If the fence is 800 square feet, how long will it take him to finish painting the fence? Round your answer to the nearest minute.

$800 = \int_0^x 200 - 4t\,dt$

$800 = 200t - 2t^2 \Big|_0^x$

$800 = 200x - 2x^2$

$x^2 - 100x + 400 = 0$



Calculator or Quadratic Formula $\quad \boxed{4.174}$

5. A particle's velocity is given by $v(t) = 2t - 8$, where t is measured in seconds, v is measured in feet per second, and $s(t)$ represents the particle's position.

(a) If $s(0) = 2$, what is the value of $s(3)$?

$2 + \int_0^3 2t - 8\,dt$

$2 + t^2 - 8t \Big|_0^3 = 2 + 9 - 24 - 0 = \boxed{-13}$

(b) What is the net change in distance over the first 5 seconds?

$\int_0^5 2t - 8\,dt$

$t^2 - 8t \Big|_0^5 = 25 - 40 - 0 = \boxed{-15 \text{ feet}}$

(c) What is the total distance traveled by the particle during the first 5 seconds? Show the set up AND your work.

$\int_0^5 |v(t)| = \left|\int_0^4 2t - 8\right| + \left|\int_4^5 2t - 8\right|$ $\qquad 2t - 8 = 0$, $t = 4$

$\left|t^2 - 8t\right| \Big|_0^4 + \left|t^2 - 8t\right| \Big|_4^5$

$|16 - 32 - 0| + |25 - 40 - (16 - 32)|$

$|-16| + |1|$

$\boxed{17 \text{ feet}}$

6. A particle's velocity is given by $v(t) = t^2 + 2t - 15$, where t is measured in minutes, v is measured in meters per minute, and $s(t)$ represents the particle's position.
(a) If $s(1) = -3$, what is the value of $s(4)$?

-12

(b) What is the net change in distance over the first 5 seconds?

$-\frac{25}{3}$ meters

(c) What is the total distance traveled by the particle during the first 5 seconds? Show the set up AND your work.

$45\frac{2}{3}$ meters or $\frac{137}{3}$ meters

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7. A particle's velocity is given by $v(t) = t^2 - 3t + 2$, where t is measured in hours, v is measured in miles per hour, and $s(t)$ represents the particle's position.
(a) If $s(2) = 5$, what is the value of $s(5)$?

$5 + \int_2^5 t^2 - 3t + 2$

$5 + \left(\frac{t^3}{3} - \frac{3}{2}t^2 + 2t\right)\Big|_2^5$

$5 + \frac{125}{3} - \frac{75}{2} + 10 - \left(\frac{8}{3} - 6 + 4\right)$

$\to 17 + \frac{117}{3} - \frac{75}{2}$

$\boxed{18.5}$

(b) What is the net change in distance over the first 5 seconds?

$\int_0^5 t^2 - 3t + 2$

$\frac{t^3}{3} - \frac{3}{2}t^2 + 2t\Big|_0^5$

$\to \frac{125}{3} - \frac{75}{2} + 10 - 0$

$\boxed{\frac{85}{6}}$

(c) What is the total distance traveled by the particle during the first 5 seconds? Show the set up AND your work.

$t^2 - 3t + 2 = 0$

$(t-2)(t-1) = 0$

$t = 1 \quad t = 2$

$\int_0^1 v(t) = \frac{5}{6}$

$\int_1^2 v(t) = -\frac{1}{6}$

$\int_2^5 v(t) = \frac{27}{2}$

$\frac{5}{6} + \frac{1}{6} + \frac{27}{2}$

$\boxed{14.5 \text{ miles}}$

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8. A particle's velocity is given by $v(t) = \cos t$, where t is measured in months, v is measured in kilometers per month, and $s(t)$ represents the particle's position.
(a) If $s\left(\frac{\pi}{6}\right) = 10$, what is the value of $s\left(\frac{3\pi}{2}\right)$?

$\boxed{8.5}$

(b) What is the net change in distance over the first π seconds?

$\boxed{0 \text{ Km}}$

(c) What is the total distance traveled by the particle during the first π seconds? Show the set up AND your work.

$\cos t = 0$ when $t = \frac{\pi}{2}$

$\boxed{2 \text{ Km}}$

9. A particle's velocity is given by $v(t) = 6\cos 3t$, where t is measured in days, v is measured in yards per day, and $s(t)$ represents the particle's position.

(a) If $s(0) = 5$, what is the value of $s\left(\frac{\pi}{2}\right)$?

$5 + \int_0^{\pi/2} v(t)\,dt = \boxed{3}$

(b) What is the net change in distance over the first $\frac{\pi}{2}$ seconds?

$\int_0^{\pi/2} V(t)\,dt = \boxed{-2 \text{ yards}}$

(c) What is the total distance traveled by the particle during the first $\frac{\pi}{2}$ seconds? Show the set up, but use a calculator to find the answer.

$\int_0^{\pi/2} |v(t)| = \boxed{6 \text{ yards}}$

10. Rate of consumption of oil in the United States during the 1980s (in billions of barrels per year) is modeled by the function

$$R(t) = 27.08e^{\frac{t}{25}}$$

where t is the number of years after January 1, 1980. Find the total consumption of oil in the United States during the 1980s.

$\int_0^{10} R(t) \approx 332.965$ billion barrels

11. $f'(x) = 6x^2 + 3$ and $f(1) = 4$. What is the value of $f(3)$?

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12. $g'(x) = \cos x$ and $g(\pi) = 7$. What is the value of $g\left(\frac{3\pi}{2}\right)$?

$7 + \int_\pi^{3\pi/2} \cos x\,dx$

$7 + \sin x \Big|_\pi^{3\pi/2}$

$7 + \sin\left(\frac{3\pi}{2}\right) - \sin(\pi)$

$7 - 1 - 0 = \boxed{6}$

13. $h'(x) = 5 - 3x$ and $h(-1) = 0$. What is the value of $h(2)$?

10.5

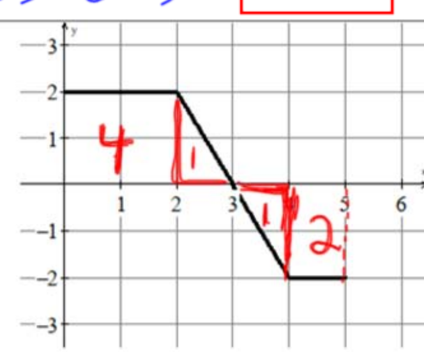
14. $f'(x) = 3x^2 - x$ and $f(2) = 6$. What is the value of $f(3)$?

$6 + \int_2^3 3x^2 - x\,dx$

$6 + \left(x^3 - \frac{1}{2}x^2\right)\Big|_2^3$

$6 + \left(27 - \frac{9}{2}\right) - (8 - 2) = \boxed{22.5}$

15. The graph to the right shows the velocity of an object moving along the x-axis over a 5-second period.



a) If the objected started 2 meters to the right, where is the object after 3 seconds?

$2 + \int_0^3 v(t) = 2 + 5 = \boxed{7}$

b) Where is the object after 5 seconds?

$2 + \int_0^5 v(t) = 2 + 5 - 3 = \boxed{4}$

c) Find the total distance traveled by the object over the 5-second period.

$\int_0^5 |v(t)|\,dt = 5 + 3 = \boxed{8}$

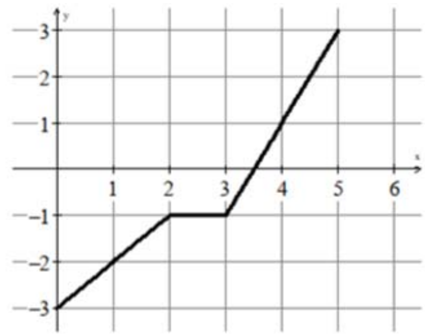
16. The graph to the right shows the velocity of an object moving along the x-axis over a 5-second period.

a) Find the total distance traveled by the object over the 5-second period.

$\boxed{7\frac{1}{2}}$

b) At time $t = 2$, the particle is at the point where $x = 4$. Where was the particle at time $t = 0$?

$\boxed{8}$



17. Fuel is pumped from an underground tank at the rates indicated in the table. The fuel is sold for \$1.50/gallon.

t (hours)	Pump rate, $P(t)$ (gals/hour)
0	250
4	300
8	400
12	500
16	200
20	650
24	450

(a) Explain the meaning of $\int_0^{24} P(t)\,dt$. Use correct units.

The number of gallons pumped over the first 24 hours.

(b) Use a left-hand Riemann sum with 6 subintervals to approximate the total amount of money earned from the sale of fuel over the 24-hour period.

$$1.5\int_0^{24} P(t)\,dt = 1.5\left[4(250+300+400+500+200+650)\right]$$
$$= \boxed{\$13{,}800}$$

Test Prep: 1C, 2C, 3D, 4B, 5D, 6D

Free Response Scoring Guide

Use this only AFTER you have attempted the problem on your own.

Solutions	Points
(a) $G'(5) = -24.588$ (or -24.587) The rate at which gravel is arriving is decreasing by 24.588 (or 24.587) tons per hour per hour at time $t = 5$ hours.	$2: \begin{cases} 1: G'(5) \\ 1: \text{interpretation with units} \end{cases}$
(b) $\int_0^8 G(t)\,dt = 825.551$ tons	$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$
(c) $G(5) = 98.140764 < 100$ At time $t = 5$, the rate at which unprocessed gravel is arriving is less than the rate at which it is being processed. Therefore, the amount of unprocessed gravel at the plant is decreasing at time $t = 5$.	$2: \begin{cases} 1: \text{compares } G(5) \text{ to } 100 \\ 1: \text{conclusion} \end{cases}$
(d) The amount of unprocessed gravel at time t is given by $A(t) = 500 + \int_0^t (G(s) - 100)\,ds$. $A'(t) = G(t) - 100 = 0 \Rightarrow t = 4.923480$ t: 0, 4.92348, 8; $A(t)$: 500, 635.376123, 525.551089 The maximum amount of unprocessed gravel at the plant during this workday is 635.376 tons.	$3: \begin{cases} 1: \text{considers } A'(t) = 0 \\ 1: \text{answer} \\ 1: \text{justification} \end{cases}$

t	$A(t)$
0	500
4.92348	635.376123
8	525.551089