Name: Solutions Date:_______________ Period: _____ **Review**

9 Review – The 2nd Fundamental Theorem of Calculus

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 9.

9.1 The 2nd FTC

Second Fundamental Theorem of Calculus

If $F(x) = \int_a^x f(t)\,dt$, where a is constant and f is a continuous function, then

$$\boldsymbol{F'(x) = f(x)}$$

If $F(x) = \int_a^{g(x)} f(t)\,dt$, where a is constant, f is a continuous function, and g is a differentiable function, then

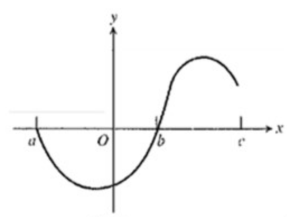
$$\boldsymbol{F'(x) = f(g(x)) \cdot g'(x)}$$

Find $F'(x)$.

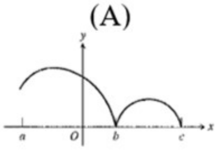
1. $F(x) = \int_0^{\sin x} t^2\,dt$	2. $F(x) = \int_{3x}^{1} \ln t\,dt$	3. $F(x) = \int_{-4}^{7x} f(t)\,dt$	4. $F(x) = \int_{x^2}^{x+8} (3t-8)\,dt$
$\sin^2 x \cos x$	$-3\ln(-3x)$ or $\ln\left(\frac{1}{27x^3}\right)$	$7f(7x)$	$[3(x+8)-8]-[3x^2-8](2x)$ $3x+24-8-6x^3+16x$ $-6x^3+19x+16$

5. Let $f(x) = \int_a^x h(t)\,dt$, where h has the graph shown below. Which of the following could be the graph of f?

D



(A)



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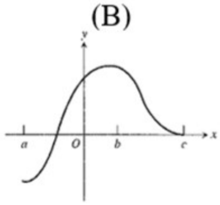

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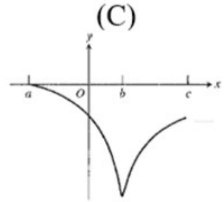

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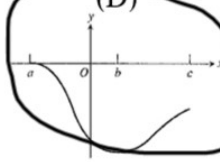
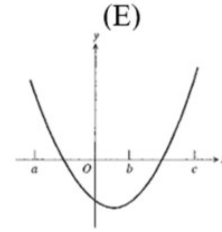


(E)



6. If $f(x) = \int_2^{\sin x} \sqrt{1+t^2}\,dt$, where $f'(x) =$

D

(A) $(1+x^2)^{\frac{3}{2}}$

(B) $\cos x\,(1+\sin x)^{\frac{1}{2}}$

(C) $(1+\sin^2 x)^{\frac{1}{2}}$

(D) $\cos x\,(1+\sin^2 x)^{\frac{1}{2}}$

(E) $\cos x\,(1+\sin^2 x)^{\frac{3}{2}}$

9.2 Trig Integrals

Trig Integrals:

$$\int \cos x\, dx = \sin x \qquad \int \sin x\, dx = -\cos x \qquad \int \sec^2 x\, dx = \tan x$$

$$\int -\csc x \cot x\, dx = \csc x \qquad \int \sec x \tan x\, dx = \sec x \qquad \int \csc^2 x\, dx = -\cot x$$

Inverse Trig Derivatives:

$\frac{d}{dx}\sin^{-1}(u) = \frac{1}{\sqrt{1-x^2}}$ | $\frac{d}{dx}\sec^{-1}(u) = \frac{1}{|x|\sqrt{x^2-1}}$ | $\frac{d}{dx}\tan^{-1}(u) = \frac{1}{x^2+1}$

For 12-15, evaluate each definite integral, indefinite integral, or derivative.

7. $\int(\cos x + \sec x \tan x)\, dx$

$\sin x + \sec x + C$

8. $\int_{\pi/4}^{\pi/3} \sec^2 x\, dx$

$\tan x \Big|_{\pi/4}^{\pi/3}$

$\tan(\frac{\pi}{3}) - \tan(\frac{\pi}{4})$

$\boxed{\sqrt{3} - 1}$

9. $\frac{d}{dx}\sin 2x$

$2\cos(2x)$

10. $\int \frac{2}{\sqrt{1-4x^2}}\, dx$

$\sin^{-1}(2x) + C$

11. $\int_{\pi/2}^{x} \cos t\, dt =$

D

(A) $-\sin x$ (B) $-\sin x - 1$ (C) $\sin x + 1$ (D) $\sin x - 1$ (E) $1 - \sin x$

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12. If the function f is defined by $f(x) = \int_0^x -\sin t^2\, dt$ on the closed interval $-1 \le x \le 3$, then f has a local maximum at $x =$

E

(A) -1.084 (B) 0 (C) 1.772 (D) 2.171 (E) 2.507

9.3 Average Value

Average Rate of Change	Mean Value Theorem	Average Value of a Function
$\dfrac{f(b)-f(a)}{b-a}$	$f'(c)=\dfrac{f(b)-f(a)}{b-a}$	$\dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$

Find the average value of each function on the given interval.

13. $f(x)=x^3$ on $[1,2]$

$\frac{1}{2-1}\int_1^2 x^3\,dx$

$\frac{1}{1}\frac{x^4}{4}\Big|_1^2$

$\frac{16}{4}-\frac{1}{4} = \boxed{\frac{15}{4}}$

14. $f(x)=\frac{1}{x}$ on $[1,e]$

$\frac{1}{e-1}\int_1^e \frac{1}{x}\,dx$

$\frac{1}{e-1}\ln x\Big|_1^e$

$\frac{1}{e-1}[\ln e-\ln 1]$

$\frac{1}{e-1}(1-0) = \boxed{\frac{1}{e-1}}$

15. The average value of the function $f(x)=(x-1)^2$ on the interval from $x=1$ to $x=5$ is

B

(A) $-\frac{16}{3}$ (B) $\frac{16}{3}$ (C) $\frac{64}{3}$ (D) $\frac{66}{3}$ (E) $\frac{256}{3}$

16. The average value of $f(x)=e^{4x^2}$ on the interval $\left[-\frac{1}{4},\frac{1}{4}\right]$ is

C

(A) 0.272 (B) 0.545 (C) 1.090 (D) 2.180 (E) 4.360

9.4 Net Change

$$\int \text{rate of change} = \text{net change}$$

$\int \text{velocity} = \text{displacement}$	$\int \lvert\text{velocity}\rvert = \text{total distance}$

17. A particle's velocity is given by $v(t) = 6t^2 - 18t + 12$, where t is measured in seconds, v is measured in feet per second, and $s(t)$ represents the particle's position.

(a) If $s(1) = 3$, what is the value of $s(2)$?

$3 + \int_1^2 v(t)\,dt = 3 + (-1) = \boxed{2}$

(b) What is the net change in distance over the first 3 seconds?

$\int_0^3 v(t)\,dt = \boxed{9 \text{ feet}}$

(c) What is the total distance traveled by the particle during the first 3 seconds? Show the set up AND your work.

$6t^2 - 18t + 12 = 0$

$t^2 - 3t + 2 = 0$

$(t-2)(t-1) = 0$

$t = 2,\ t = 1$

$\int_0^1 v(t) + \int_1^2 v(t) + \int_2^3 v(t)$

$\boxed{11 \text{ feet}}$

18. If $f'(x) = -\cos x$ and $f(\pi/4) = -1$. What is the value of $f\left(\frac{\pi}{2}\right)$?

$-1 + \int_{\pi/4}^{\pi/2} (-\cos x)\,dx$

$-1 + (-\sin x)\Big|_{\pi/4}^{\pi/2}$

$-1 + \left[-\sin\frac{\pi}{2} - -\sin\frac{\pi}{4}\right]$

$-1 + \left[-(1) + \left(\frac{\sqrt{2}}{2}\right)\right]$

$\boxed{-2 + \frac{\sqrt{2}}{2}}$

19. Find the distance traveled (to three decimal places) from $t = 1$ to $t = 5$ seconds, for a particle whose velocity is given by $v(t) = t + \ln t$.

C

(A) 6.000 (B) 1.609 (C) 16.047 (D) 0.800 (E) 148.413

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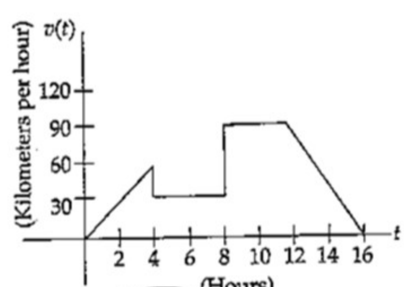
20. A car's velocity is shown on the graph above. Which of the following gives the total distance traveled from $t = 0$ to $t = 16$ (in kilometers)?

C



(A) 360 (B) 390 (C) 780 (D) 1000 (E) 1360