UNIT 10 More Integration NAME: Key

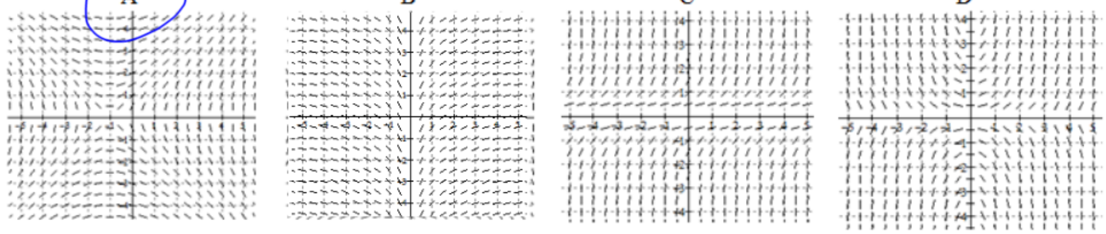
REVIEW

when the numerator equals zero the slope is zero

1. Which of the following is the slope field of $\frac{dy}{dx} = \frac{x+1}{y}$?

when the denominator equals zero the slope is undefined (nothing or vertical)

A B C D



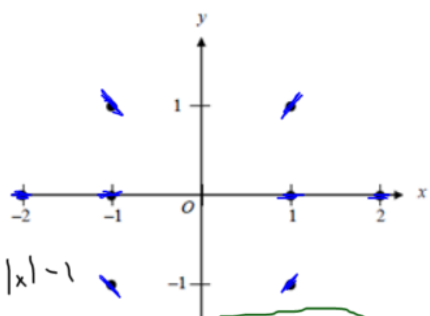
A

$y=0$ has undefined slope

$x=-1$ has zero slope

2. Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x}$, where $x \neq 0$.

   a. On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.



   b. Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$.

$\frac{1}{y^2}dy = \frac{1}{x}dx$

$\int \frac{1}{y^2}dy = \int \frac{1}{x}dx$

$-y^{-1} = \ln|x| + c$

$-\frac{1}{1} = \ln|-1| + c$

$-1 = \ln 1 + c$

$-1 = 0 + c$

$-1 = c$

$-\frac{1}{y} = \ln|x| - 1$

$\frac{1}{y} = -\ln|x| + 1$

$y = \frac{-1}{\ln|x| + 1}$

   c. Write an equation for the tangent line to the curve $y = f(x)$ through the point (1, -1). Then use your tangent line equation to estimate the value of $f(1.2)$.

slope of the tangent line at (1,-1)= $\frac{dy}{dx} = \frac{(-1)^2}{1} = 1$

$y + 1 = 1(x - 1)$

$y = x - 2$ equation of the tangent line at (1,-1)

so $y = 1.2 - 2$

$y = -0.8$ estimate of f(1.2)

Find the indefinite integrals.

3. $\int x^5 \sin(x^6 + 2)dx$

$u = x^6 + 2$

$du = 6x^5 dx$

$\frac{1}{6}du = x^5 dx$

$\frac{1}{6}\int \sin(u)\,du$

$\frac{1}{6}(-\cos(u) + c)$

$-\frac{1}{6}\cos(x^6 + 2) + c$

4. $\int (x+1)\sqrt{x^2 + 2x}\,dx$

$u = x^2 + 2x$

$du = 2x + 2\,dx$

$du = 2(x+1)\,dx$

$\frac{1}{2}du = x + 1\,dx$

$\frac{1}{2}\int \sqrt{u}\,du$

$\frac{1}{2}\left(\frac{2}{3}u^{3/2} + c\right)$

$\frac{1}{3}\sqrt{(x^2+2x)^3} + c$

5. $\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}dx$

$2\,du = \frac{1}{\sqrt{x}}dx$

$2\int \cos(u)\,du$

$2(\sin(u) + c)$

$2\sin\sqrt{x} + c$

Evaluate the definite integrals.

6. $\int_{-1}^{1} x\sqrt{1-x^2}\,dx$

$u = 1 - x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$-\frac{1}{2}\int_0^0 \sqrt{u}\,du = 0$

7. $\int_0^{\frac{\pi}{6}} \frac{\sin(2x)}{\cos^2(2x)}\,dx$

x	u
0	1
$\frac{\pi}{6}$	1/2

$u = \cos(2x)$

$du = -2\sin(2x)\,dx$

$-\frac{1}{2}du = \sin(2x)\,dx$

$-\frac{1}{2}\int_1^{1/2} \frac{1}{u^2}\,du = \frac{1}{2}\int_{1/2}^{1} u^{-2}\,du$

$\frac{1}{2}\left[-u^{-1}\right]_{1/2}^{1}$

$\frac{1}{2}\left[\left(-\frac{1}{1}\right) - \left(-\frac{1}{1/2}\right)\right]$

$\frac{1}{2}(-1+2)$

$\frac{1}{2}$

8. $\int_e^{e^2} \frac{1}{x\ln x}\,dx$

x	u
e	1
e^2	2

$u = \ln x$

$du = \frac{1}{x}dx$

$\int_1^2 \frac{1}{u}\,du$

$\ln|u|\Big]_1^2$

$\ln 2 - \ln 1$

$\ln 2 - 0$

$\ln 2$

TEST PREP

use u substitution!!!

1. If $\int_1^4 f(x)dx = 6$, what is the value of $\int_1^4 f(5-x)dx$?

$-\int_4^1 f(u)\,du = \int_1^4 f(u)\,du = 6$

(A) 6
(B) 3
(C) 0
(D) -1
(E) -6

A

$u = 5 - x$

$du = -dx$

$-du = dx$

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2. $\int \frac{e^{x^2}-2x}{e^{x^2}}\,dx =$

seperate the fraction!

(A) $x - e^{x^2} + C$
(B) $x - e^{-x^2} + C$
(C) $x + e^{x^2} + C$
(D) $x + e^{-x^2} + C$
(E) $e^{x^2} + C$

D

$\int \frac{e^{x^2}}{e^{x^2}}\,dx - \int \frac{2x}{e^{x^2}}\,dx$

$\int 1\,dx - \int \frac{2x}{e^{x^2}}\,dx$

$x + c$

$u = x^2$

$du = 2x\,dx$

$\int \frac{1}{e^u}\,du = \int e^{-u}\,du$

$-e^{-u} + c$

$-e^{-x^2} + c$

$x - (-e^{-x^2} + c)$

$x + e^{-x^2} + c$

3. $\displaystyle\int_{\pi/4}^{\pi/3} \frac{\sec^2 x}{\tan x}\,dx =$

(A) $\ln\sqrt{3}$
(B) $-\ln\sqrt{3}$
(C) $\ln 2$
(D) $\sqrt{3}-1$
(E) $\ln\frac{\pi}{3} - \ln\frac{\pi}{4}$

A

$u = \tan x$

$du = \sec^2 x\,dx$

x	u
$\pi/4$	1
$\pi/3$	$\sqrt{3}$

$\int_1^{\sqrt{3}} \frac{1}{u}\,du$

$\ln|u|\Big]_1^{\sqrt{3}}$

$\ln\sqrt{3} - \ln 1$

$\ln\sqrt{3} - 0$

$\ln\sqrt{3}$

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4. $\displaystyle\int_0^5 \frac{dx}{\sqrt{1+3x}} =$

(A) 4
(B) $\frac{8}{3}$
(C) 2
(D) $\frac{16}{5}$
(E) 1

C

$u = 1+3x$

$du = 3\,dx$

$\frac{1}{3}du = dx$

x	u
0	1
5	16

$\frac{1}{3}\int_1^{16} \frac{1}{\sqrt{u}}\,du = \frac{1}{3}\int_1^{16} u^{-1/2}\,du$

$\frac{1}{3}\left[2u^{1/2}\right]_1^{16}$

$\frac{1}{3}\left[2\sqrt{16} - 2\sqrt{1}\right]$

$\frac{1}{3}\left[8-2\right]$

$\frac{1}{3}(6) = 2$

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5. $\displaystyle\int_0^4 \frac{2x}{x^2+9}\,dx =$

(A) 25
(B) 16
(C) $\ln\frac{25}{9}$
(D) $\ln 4$
(E) $\ln\frac{8}{3}$

C

$u = x^2+9$

$du = 2x\,dx$

x	u
0	9
4	25

$\int_9^{25} \frac{1}{u}\,du = \left[\ln|u|\right]_9^{25}$

$\ln 25 - \ln 9$

$\ln\frac{25}{9}$

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6. $\displaystyle\int \frac{e^{\sqrt{x}}}{2\sqrt{x}}\,dx =$

(A) $\ln\sqrt{x} + C$
(B) $x + C$
(C) $e^x + C$
(D) $\frac{1}{2}e^{2\sqrt{x}} + C$
(E) $e^{\sqrt{x}} + C$

E

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$\int e^u\,du$

$e^u + C$

$e^{\sqrt{x}} + C$

7. If the substitution $u = \frac{x}{2}$ is made, the integral $\int_2^4 \frac{1+\left(\frac{x}{2}\right)^2}{x}\,dx = \cancel{2}\int_1^2 \frac{1+u^2}{\cancel{2}u} = \int_1^2 \frac{1+u^2}{u}$

(A) $\int_1^2 \frac{1+u^2}{u}\,du$

(B) $\int_2^4 \frac{1+u^2}{u}\,du$

(C) $\int_1^2 \frac{1-u^2}{2u}\,du$

(D) $\int_1^2 \frac{1-u^2}{4u}\,du$

(E) $\int_2^4 \frac{1-u^2}{2u}\,du$

$u = \frac{x}{2}$ → $2 \cdot u = \frac{x}{2} \cdot 2$

$du = \frac{1}{2}\,dx$

$2\,du = dx$ $\quad \boxed{2u = x}$

x	u
2	1
4	2

NOTE: This will not be on the Unit 10 Test but you might see one on the AP exam!

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8. $\int_0^{\pi/4} \frac{e^{\tan x}}{\cos^2 x}\,dx =$

REWRITE! $\int_0^{\pi/4} e^{\tan x} \frac{1}{\cos^2 x}\,dx = \int_0^{\pi/4} e^{\tan x} \sec^2 x\,dx = \int_0^1 e^u\,du$

(A) 0

(B) 1

(C) $e-1$

(D) e

(E) $e+1$

C

$u = \tan x$

$du = \sec^2 x\,dx$

x	u
0	0
$\pi/4$	1

$e^u\Big]_0^1$

$e^1 - e^0$

$e^1 - 1$

$e - 1$

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9. $\int_0^1 x^3 e^{x^4}\,dx =$

(A) $\frac{1}{4}(e-1)$

(B) $\frac{1}{4}e$

(C) $e-1$

(D) e

(E) $4(e+1)$

A

$u = x^4$

$du = 4x^3\,dx$

$\frac{1}{4}\,du = x^3\,dx$

$\frac{1}{4}\int_0^1 e^u\,du$

$\frac{1}{4}\left[e^u\right]_0^1$

$\frac{1}{4}\left[e^1 - e^0\right]$

$\frac{1}{4}\left[e^1 - 1\right]$

$\frac{1}{4}(e-1)$

x	u
0	0
1	1

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10. $\int_1^2 \frac{x+1}{x^2+2x}\,dx =$

(A) $\ln 8 - \ln 3$

(B) $\frac{\ln 8 - \ln 3}{2}$

(C) $\ln 8$

(D) $\frac{3\ln 2}{2}$

(E) $\frac{3\ln 2 + 2}{2}$

B

$u = x^2 + 2x$

$du = (2x+2)\,dx$

$du = 2(x+1)\,dx$

$\frac{1}{2}\,du = (x+1)\,dx$

$\frac{1}{2}\int_3^8 \frac{du}{u}$

$\frac{1}{2}\left[\ln|u|\right]_3^8$

$\frac{1}{2}(\ln 8 - \ln 3)$

x	u
1	3
2	8