Name: Solutions Date:_______________ Period: _____

Review

1 Review – Limits

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 1.

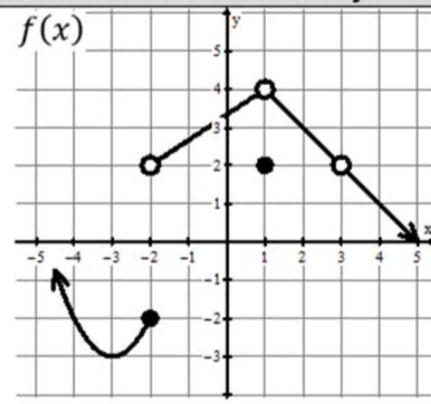
1.1 Limits Graphically:

What is a limit?

The ***y*-value** a function approaches at a given x-value.

Give the value of each statement. If the value does not exist, write "*does not exist*" or "*undefined*."

1. $\lim_{x\to 3} f(x) =$ 2
2. $\lim_{x\to 1} f(x) =$ 4
3. $f(3) =$ DNE
4. $f(-2) =$ -2
5. $\lim_{x\to 2} f(x) =$ 3
6. $\lim_{x\to -2^+} f(x) =$ 2
7. $f(1) =$ 2
8. $\lim_{x\to -2^-} f(x) =$ -2



1.2 Limits Analytically:

Finding a limit:

1. Direct Substitution.
2. Simplify and then try direct substitution.
   a. Factor and Cancel.
   b. Rationalize if you see square roots.
3. L'Hôpital's Rule (for indeterminate forms $\frac{0}{0}$ or $\frac{\infty}{\infty}$)

Special Trig Limits:

$$\lim_{x\to 0}\frac{\sin x}{x} = 1 \quad \text{or} \quad \lim_{x\to 0}\frac{x}{\sin x} = 1$$

$$\lim_{x\to 0}\frac{1-\cos x}{x} = 0 \quad \text{or} \quad \lim_{x\to 0}\frac{\cos x - 1}{x} = 0$$

Evaluate each limit.

9. $\lim_{x\to -4}(2x^2+3x-2)$ $2(-4)^2+3(-4)-2$ $32-12-2$ $\boxed{18}$	10. $\lim_{x\to 1}\sqrt{7x+42}$ $\sqrt{7+42}$ $\sqrt{49}$ $\boxed{7}$	11. $\lim_{x\to 13} 2$ $\boxed{2}$	12. $\lim_{x\to 10}\frac{x^2-5x-50}{x-10}$ $\frac{\cancel{(x-10)}(x+5)}{\cancel{x-10}}$ $10+5=\boxed{15}$

13. $\lim_{x\to 0} \frac{\sqrt{x+19}-\sqrt{19}}{x} \cdot \frac{\sqrt{x+19}+\sqrt{19}}{\sqrt{x+19}+\sqrt{19}} = \lim_{x\to 0} \frac{(x+19)-(19)}{x(\sqrt{x+19}+\sqrt{19})}$

$\lim_{x\to 0} \frac{\cancel{x}}{\cancel{x}(\sqrt{x+19}+\sqrt{19})} = \frac{1}{\sqrt{0+19}+\sqrt{19}} = \frac{1}{2\sqrt{19}} = \boxed{\frac{\sqrt{19}}{38}}$

14. $\lim_{x\to 0} \frac{\frac{1}{x+1}-1}{x} \cdot \frac{x+1}{x+1} = \lim_{x\to 0} \frac{1-(x+1)}{x(x+1)}$

$\lim_{x\to 0} \frac{-\cancel{x}}{\cancel{x}(x+1)} = \lim_{x\to 0} \frac{-1}{0+1} = \boxed{-1}$

15. $\lim_{x\to 0} \frac{\sin(7x)}{11x} = \lim_{x\to 0} \frac{1}{11} \cdot \frac{\sin(7x)}{x} \cdot \frac{7}{7}$

$\lim_{x\to 0} \frac{7}{11} \cdot \frac{\sin(7x)}{7x} = \boxed{\frac{7}{11}}$

16. $\lim_{x\to 0} \frac{\sin^2(3x)}{\sin^2(5x)}$

$\lim_{x\to 0} \frac{\sin(3x)}{3x} \frac{\sin(3x)}{3x} \frac{5x}{\sin(5x)} \frac{5x}{\sin(5x)} \cdot \frac{3}{5} \cdot \frac{3}{5} = \boxed{\frac{9}{25}}$

1.3 Asymptotes:

Vertical Asymptotes:

If the denominator equals 0, then there is a hole or a vertical asymptote. If the factor does not cancel, then it's a vertical asymptote.

One-sided limits at vertical asymptotes approach $-\infty$ or ∞.

Horizontal asymptotes:

$\lim_{x\to\infty} \frac{f(x)}{g(x)}$ will produce a horizontal asymptote at

- $y = 0$ if g increases faster than f.
- $y = \frac{a}{b}$ if g and f are increasing at the relative same amount where a and b are the coefficients of the fastest growing terms.

Don't forget to check the left and right sides when looking for horizontal asymptotes.

Evaluate each limit.			Find all horizontal asymptotes.
17. $\lim_{x\to\infty} \frac{4x^5-2x^2+3}{3x^2+2x^5-x^4}$ $\frac{4}{2} = \boxed{2}$	18. $\lim_{x\to\infty} x^5 3^{-x}$ $\lim_{x\to\infty} \frac{x^5}{3^x}$ BIG! $\boxed{0}$	19. $\lim_{x\to\infty} \sin \frac{x+3\pi x^2}{2x^2}$ $\sin\left(\frac{3\pi}{2}\right)$ $\boxed{-1}$	20. $f(x) = \frac{\sqrt{16x^6+x^3+5x}}{5x^3-8x}$ as $x\to\infty$, $y=4/5$ as $x\to-\infty$, $y=-4/5$

1.4 Continuity:

Types of Discontinuities:

1. Removable (hole).
2. Discontinuity due to vertical asymptote.
3. Jump discontinuity.

Finding Domain:

Restrictions occur with two scenarios:

1. Denominators can't be zero.
2. Even radicals can't be negative.

Don't forget the Intermediate Value Theorem (for continuous functions)! What is it and what does it tell us?