TEST PREP

$f(x) = [\sin x]^2$

1. What are the x-coordinate(s) of the points of inflection for the graph of $f(x) = \sin^2 x$ on the closed interval $[0, \pi]$?

   (A) $x = \frac{3\pi}{4}$ only
   (B) $x = \frac{\pi}{4}$, $x = \frac{\pi}{2}$, and $x = \frac{3\pi}{4}$
   (C) $x = \frac{\pi}{4}$ and $x = \frac{3\pi}{4}$
   (D) $x = \frac{\pi}{2}$ only
   (E) $x = \frac{\pi}{4}$ only

C

$2\cos^2 x = 2\sin^2 x$

true when cos and sin are the same!

$\frac{3\pi}{4}$, $\frac{\pi}{4}$, π, 0

$f'(x) = 2(\sin x)(\cos x)$

$u \quad v$

$u = \sin x \qquad v = \cos x$

$u' = \cos x \qquad v' = -\sin x$

$f''(x) = 2[\cos x \cdot \cos x + \sin x(-\sin x)]$

$f''(x) = 2[\cos^2 x - \sin^2 x]$

$2\cos^2 x - 2\sin^2 x = 0$

2. The function defined by $g(x) = 4x^3 - 3x^2$ for all values of x has a relative maximum at $x =$

   (A) $-\frac{1}{2}$
   (B) 0
   (C) $\frac{1}{2}$
   (D) $\frac{1}{4}$
   (E) 1

B

$g'(x) = 12x^2 - 6x$

$6x(2x-1) = 0$

$x = 0, \frac{1}{2}$

	-1	0	$\frac{1}{4}$	$\frac{1}{2}$	1
$f'(x)$	$+$	0	$-$	0	$+$

max min

3. The graph of the derivative of function f is shown below. At what value of x does function f have a relative maximum?

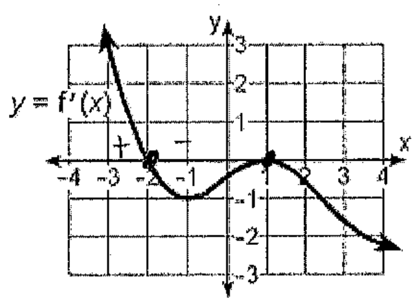
   (A) 1
   (B) -1
   (C) 0
   (D) -2
   (E) 3

D

$f'(-2) = 0$

Changes from $+$ to $-$

max



4. The function g is defined by the equation $g(x) = 6x^5 - 10x^3$. Determine the values of x for which the graph of function g is concaved upwards.

   (A) $x > \frac{1}{2}$
   (B) $-\frac{\sqrt{2}}{2} < x < 0$ or $x > \frac{\sqrt{2}}{2}$
   (C) $-\frac{1}{2} < x < 0$ or $x > \frac{1}{2}$
   (D) $-\frac{1}{2} < x < \frac{1}{2}$
   (E) $-\frac{\sqrt{2}}{2} < x < \frac{\sqrt{2}}{2}$

B

$g'(x) = 30x^4 - 30x^2$

$g''(x) = 120x^3 - 60x$

$60x(2x^2 - 1) = 0$

$x = 0, \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$

	-1	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	1
$f''(x)$	$-$	0	$+$	0	$-$	0	$+$

concave up

$f(x) = x^2 + kx^{-1}$ $\qquad$ $f'(2) = 0$

5. For what values of k will $f(x) = x^2 + \frac{k}{x}$ have a relative minimum at $x = 2$?

   (A) -2

   (B) 2

   (C) 8

   (D) -16

   (E) 16

E

$f'(x) = 2x - kx^{-2}$

$2x - \frac{k}{x^2} = 0$

$2(2) - \frac{k}{2^2} = 0$

$4 - \frac{k}{4} = 0$

$4 = \frac{k}{4}$

$16 = k$

6. The graph shown below shows the derivative f' of the function f. At what value(s) of x does function f have a point of inflection?

looking for $f'' = 0$

   (A) c and e only

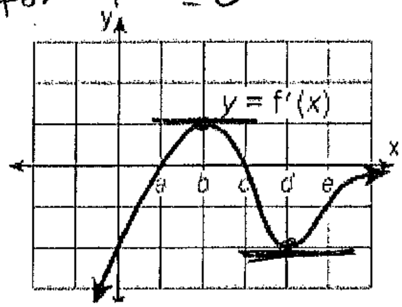
   (B) a, b, c, and d only

   (C) a and c only

   (D) b and d only

   (E) a only

D



7. An equation of the line tangent to the graph of $f(x) = 2x^3 - 3x^2$ at its point of inflection is

   (A) $3x + 2y = 5$

   (B) $6x + 4y = 1$

   (C) $6x + 4y = 5$

   (D) $3x + 2y = 1$

   (E) $6x - 4y = 1$

B

$f'(x) = 6x^2 - 6x$

$f''(x) = 12x - 6$

$12x - 6 = 0$

$x = \frac{6}{12}$

$\boxed{x = \frac{1}{2}\ \text{POI}}$

$f(\tfrac{1}{2}) = 2(\tfrac{1}{2})^3 - 3(\tfrac{1}{2})^2$

$f(\tfrac{1}{2}) = \frac{1}{4} - \frac{3}{4}$

$f(\tfrac{1}{2}) = -\frac{1}{2}$

$f'(\tfrac{1}{2}) = 6(\tfrac{1}{2})^2 - 6(\tfrac{1}{2})$

$\frac{3}{2} - \frac{6}{2}$

$f'(\tfrac{1}{2}) = -\frac{3}{2}$

$y + \frac{1}{2} = -\frac{3}{2}(x - \frac{1}{2})$

$4[y + \frac{1}{2} = -\frac{3}{2}x + \frac{3}{4}]$

$4y + 2 = -6x + 3$

$6x + 4y = 1$

8. The graph of the function $y = f(x)$ is shown below. On which of the following intervals is $f'(x) > 0$ and $f''(x) > 0$?

concave up!

positive tangent

   (A) I, II, and III

   (B) II and III only

   (C) II only

   (D) I only

   (E) III only

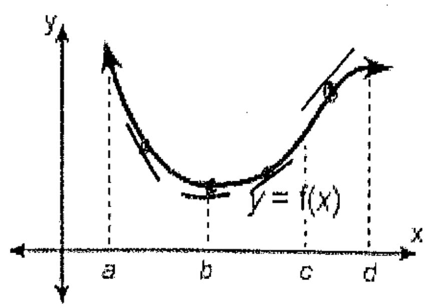
E

I. $c < x < d$

II. $a < x < b$

III. $b < x < c$

positive tangent and $f(x)$ is concave up



9. The graph of the derivative of function f is shown below. Where on the interval $[-2, 3]$ is function f decreasing? $\rightarrow$ $f'(x) < 0$

(A) $[-2, 3]$

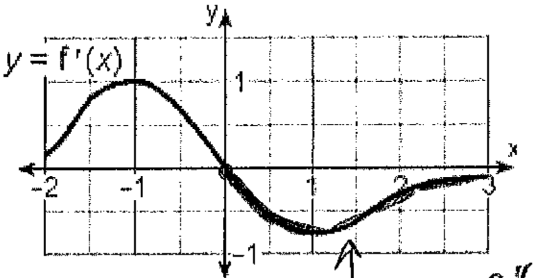
(B) $[-1, 1]$

(C) $[1, 3]$

(D) $[-2, 0]$

(E) $[0, 3]$

E



10. For what interval is $f(x) = \frac{1}{1-x^2}$ increasing? $x \neq \pm 1$

(A) Function f increases for all real values of x

(B) $(-\infty, -1) \cup (-1, 0]$

(C) $[0, 1) \cup (1, \infty)$

(D) $(-1, 1)$

(E) $(-\infty, -1) \cup (1, \infty)$

C

when is $f'(x) > 0$

$f(x) = (1-x^2)^{-1}$

$f'(x) = -1(1-x^2)^{-2}(-2x)$

$\frac{2x}{(1-x^2)^2} = 0$

$x = 0$ $\quad$ $x \neq \pm 1$ Not in domain of $f(x)$

$f'(x)$: $(-\infty, -1)$ $-$ $\quad(-1, 0)$ $-$ $\quad(0, 1)$ $+$ $\quad(1, \infty)$ $+$

11. The table below shows various values for the derivatives of differentiable functions f, g, and h. Which of these functions must have a relative maximum on the open interval $(-3, 3)$?

(A) g only

(B) f, g, and h

(C) g and h only

(D) h only

(E) f only

D

x	-3	-2	-1	0	1	2	3	
$f'(x)$	0.5	1	1.5	2	1.5	1	0.5	X
$g'(x)$	-1.5	-1	-0.5	0	0.5	1	1.5	X
$h'(x)$	-0.5	0	-0.5	0	0.5	0	-0.5	

Max

12. If $\boxed{\lim_{h\to 0} \frac{f(-2+h) - f(-2)}{h}} = 2.637$, then the graph of function f at $x = -2$ is

(A) Decreasing

(B) Concave downwards

(C) Increasing

(D) Concave upwards

(E) Stationary

C

Means $f'(-2)$

$f'(-2) = 2.637$ positive

13. The graph of $y = f(x)$ is shown below. If f is twice-differentiable, which of the following is true?

(A) $\cancel{f(x)} < 0$, $f'(x) < 0$, $f''(x) < 0$

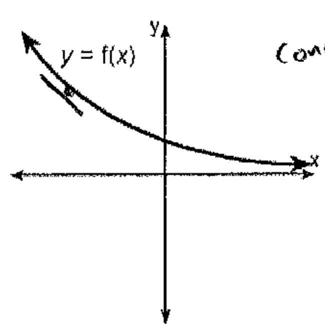
(B) $f(x) > 0$, $f'(x) < 0$, $f''(x) > 0$

(C) $f(x) > 0$, $f'(x) > 0$, $f''(x) > 0$

(D) $f(x) > 0$, $f'(x) < 0$, $f''(x) < 0$

(E) $f(x) > 0$, $f'(x) > 0$, $f''(x) < 0$

B



$f(x) > 0$

$f'(x) < 0$

$f''(x) > 0$

Concave up so $f''(x) =$ positive