3.5 Selecting Procedures for Determining Derivatives

Calculus

Solutions **Practice**

1. If $f(x) = x^2 \ln x$, then $f'(x) =$

$f'(x) = 2x \ln x + x^2 \cdot \frac{1}{x}$

(A) 2 (B) $x + 2\ln x$ (C) $2x \ln x$ (D) $1 + 2x\ln x$ (E) $x + 2x \ln x$

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2. If f and g are functions such that $f(g(x)) = x$ for all x in their domains, and if $f(a) = b$ and $f'(a) = c$, then which of the following is true? → means g is inverse of f

$g'(b) = \frac{d}{dx} f^{-1}(b) = \frac{1}{f'(f^{-1}(b))} = \frac{1}{f'(a)} = \frac{1}{c}$

(A) $g'(a) = \frac{1}{c}$

(B) $g'(a) = -\frac{1}{c}$

(C) $g'(b) = \frac{1}{c}$

(D) $g'(b) = -\frac{1}{c}$

(E) $g'(b) = \frac{1}{a}$

3. Find the equation of the tangent line to $9x^2 + 16y^2 = 52$ though $(2, -1)$.

$18x + 32y\frac{dy}{dx} = 0$ $\quad 8 \cdot (y+1) = \frac{9}{8}(x-2) \cdot 8$

$36 - 32\frac{dy}{dx} = 0$ $\quad 8y + 8 = 9x - 18$

$0 = 9x - 8y = -26$

$\frac{dy}{dx} = \frac{9}{8}$

(A) $-9x + 8y - 26 = 0$ (B) $9x - 8y - 26 = 0$ (C) $9x - 8y - 106 = 0$

(D) $8x + 9y - 17 = 0$ (E) $9x + 16y - 2 = 0$

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4. What is the slope of the line tangent to the curve $y = \arctan(2x)$ at the point when $x = \frac{1}{2}$?

$y' = \frac{1}{4x^2+1} \cdot 2$

$y'(\frac{1}{2}) = \frac{1}{1+1} \cdot 2 = 1$

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 4

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5. If $f(x) = \frac{3x^2+x}{3x^2-x}$ then $f'(x)$ is

Quotient Rule!

$f'(x) = \frac{(6x+1)(3x^2-x) - (3x^2+x)(6x-1)}{(3x^2-x)^2} = 18x^3 - 6x^2 + 3x^2 - x - (18x^3 - 3x^2 + 6x^2 - x)$

$\frac{-6x^2}{x^2(3x-1)^2}$

(A) 1 (B) $\frac{6x^2+1}{3x^2-x}$ (C) $\frac{-6}{(3x-1)^2}$

(D) $\frac{-2x^2}{(x^2-x)^2}$ (E) $\frac{36x^2-2x}{(x^2-x)^2}$

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6. If $f(x) = \sqrt{1+\sqrt{x}}$, find $f'(x)$. $\frac{1}{2\sqrt{1+\sqrt{x}}} \cdot \left(\frac{1}{2\sqrt{x}}\right)$

(A) $\frac{-1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$ (B) $\frac{1}{2\sqrt{x}\sqrt{1+\sqrt{x}}}$ (C) $\frac{1}{4\sqrt{1+\sqrt{x}}}$

(D) $\frac{1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$ (E) $\frac{-1}{2\sqrt{x}\sqrt{1+\sqrt{x}}}$

7. A curve is generated by the equation $x^2 + 4y^2 = 16$. Determine the number of points on this curve whose corresponding tangent lines are horizontal.

$$2x + 8y\frac{dy}{dx} = 0$$
$$8y\frac{dy}{dx} = -2x$$
$$\frac{dy}{dx} = \frac{-x}{4y} \to 0 \text{ when } x = 0$$

$$0^2 + 4y^2 = 16$$
$$y^2 = 4$$
$$y = \pm 2$$

Two points!

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

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8. $\frac{d}{dx}(\underbrace{\ln(3x)}_{f}\ \underbrace{5^{2x}}_{g}) =$

Product rule!

$$\frac{1}{3x}\cdot 3 \cdot 5^{2x} + \ln(3x)\, 5^{2x} \ln(5)\cdot 2$$
$$\frac{5^{2x}}{x} + 2\ln(5)\ln(3x)\, 5^{2x}$$

(A) $\frac{5^{2x}}{x} + 2\ln(5)\ln(3x)\,5^{2x}$ (B) $\frac{5^{2x}}{3x} - 2x\ln(3x)\,5^{2x}$ (C) $\frac{5^{2x}}{x} - \ln(5)\ln(3x)\,5^{2x}$

(D) $\frac{5^{2x}}{3x} + 2\ln(3x)5^{2x}$ (E) $\frac{5^{2x}}{x} + \ln(5)\ln(3x)\,5^{2x}$

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9. Let a function f be defined as $f(x) = x^3 - 2x - 4$ for $x \geq 1$. Let $g(x)$ be the inverse function of $f(x)$ and note that $f(2) = 0$. The value of $g'(0) =$

$$g'(0) = \frac{1}{f'(f^{-1}(0))} = \frac{1}{f'(2)}$$

$$f'(x) = 3x^2 - 2$$
$$f'(2) = 12 - 2 = 10$$

(A) $\frac{1}{10}$ (B) 1 (C) 4 (D) -2 (E) $-\frac{1}{2}$

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10. $\frac{d}{dx}\left(\sin^{-1} x + 2\sqrt{x}\right) =$ $\frac{1}{\sqrt{1-x^2}} + 2\cdot\frac{1}{2\sqrt{x}}$

(A) $-\frac{1}{\sin^2 x} + \frac{1}{\sqrt{x}}$ (B) $\frac{1}{\sqrt{1-x^2}} + 4\sqrt[3]{x}$ (C) $\frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{x}}$

(D) $\frac{1}{\sqrt{x^2-1}} + 4\sqrt[3]{x}$ (E) $\frac{1}{\sqrt{x^2-1}} + \frac{1}{\sqrt{x}}$