6.8 Indefinite Integrals

Calculus

Solutions

Practice

Find the following indefinite integrals.

1. $\int \left(6^x - \frac{1}{x}\right) dx$

$$\frac{1}{\ln 6} 6^x - \ln|x| + C$$

2. $\int \left(\frac{x^5-6}{x}\right) dx$

$$\int x^4 - \frac{6}{x} \, dx$$

$$\frac{1}{5} x^5 - 6\ln|x| + C$$

3. $\int (e^x + e^2) \, dx$

$$e^x + e^2 x + C$$

4. $\int 5^x \, dx$

$$\frac{1}{\ln 5} 5^x + C$$

5. $\int \frac{1}{x \cdot \sqrt[3]{x}} \, dx$

$$\int \frac{1}{x^{4/3}} \, dx = \int x^{-4/3} \, dx$$

$$-3 x^{-1/3} + C$$

6. $\int (3-x)^2 \, dx$

$$\int (9 - 6x + x^2) \, dx$$

$$9x - 3x^2 + \frac{1}{3}x^3 + C$$

7. $\int \sqrt{t} \left(t - \frac{1}{t}\right) dt$

$$\int t^{3/2} - t^{-1/2} \, dt$$

$$\frac{2}{5} t^{5/2} - 2t^{1/2} + C$$

8. $\int \left(\frac{5x^2+x-2}{x}\right) dx$

$$\int \left(5x + 1 - \frac{2}{x}\right) dx$$

$$\frac{5}{2} x^2 + x - 2\ln|x| + C$$

9. $\int (x - 2\csc^2 x) \, dx$

$$\frac{1}{2}x^2 + 2\cot x + C$$

10. $\int (x^2 + 2)^2 \, dx$

$$\int (x^4 + 4x^2 + 4) \, dx$$

$$\frac{1}{5} x^5 + \frac{4}{3}x^3 + 4x + C$$

11. $\int (3\csc x \cot x - 1) \, dx$

$$-3\csc x - x + C$$

12. $\int \left(\frac{\sqrt{x}-x-5}{x}\right) dx$

$$\int \left(x^{-1/2} - 1 - \frac{5}{x}\right) dx$$

$$2\sqrt{x} - x - 5\ln|x| + C$$

13. $\int (5 - \sec^2 x) \, dx$

$$5x - \tan x + C$$

14. $\int (3\sin x - \sqrt{x}) \, dx$

$$-3\cos x - \frac{2}{3} x^{3/2} + C$$

Find the function that satisfies the given conditions.

15. $h'(t) = 8t^3 + 5$ and $h(1) = -4$

$$h(t) = \frac{8t^4}{4} + 5t + C$$

$$-4 = 2(1)^4 + 5(1) + C$$

$$-4 = 7 + C$$

$$-11 = C$$

$$h(t) = 2t^4 + 5t - 11$$

16. $\frac{dy}{dx} = 2x + \sin x$ and $y(0) = 4$

$$y = x^2 - \cos x + C$$

$$4 = 0 - \cos(0) + C$$

$$4 = -1 + C$$

$$5 = C$$

$$y = x^2 - \cos x + 5$$

17. $f''(x) = x^{-3/2}$ and $f'(4) = 2$ and $f(0) = 0$

$f'(x) = -2x^{-\frac{1}{2}} + C$

$2 = -2\frac{1}{\sqrt{4}} + C$

$3 = C$

$f'(x) = -2x^{-\frac{1}{2}} + 3$

$f(x) = -4x^{\frac{1}{2}} + 3x + C$

$0 = C$

$\boxed{f(x) = -4\sqrt{x} + 3x}$

18. $f''(x) = \sin x$ and $f'(0) = 1$ and $f(0) = 6$

$f'(x) = -\cos x + C$

$1 = -\cos(0) + C$

$2 = C$

$f'(x) = -\cos x + 2$

$f(x) = -\sin x + 2x + C$

$6 = -\sin(0) + 0 + C$

$6 = C$

$\boxed{f(x) = -\sin x + 2x + 6}$

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The following problems are DEFINITE integrals, but use strategies that were covered in this lesson.

19. $\int_1^3 \frac{x+6}{x^2} dx = \int_1^3 \frac{1}{x} + 6x^{-2} = \left[\ln|x| - 6x^{-1} \right] \Big|_1^3$

$\left[\ln(3) - \frac{6}{3} \right] - \left[\ln 1 - \frac{6}{1} \right]$

$(\ln 3 - 2) - (0 - 6)$

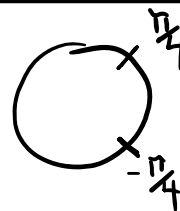(A) $-\frac{1}{3}$ (B) $\ln 3 + 4$ (C) $\ln 3$ (D) 3 (E) $\ln 3 - 8$

20. $\int_{-1}^1 \frac{4}{1+x^2} dx = 4\tan^{-1}x \Big|_{-1}^1$

$4\tan^{-1}(1) - 4\tan^{-1}(-1)$

$4\left(\frac{\pi}{4}\right) - 4\left(-\frac{\pi}{4}\right)$

$\pi + \pi$

(A) 0 (B) π (C) 1 (D) 2π (E) 2