Calculus

1.11 Defining Continuity at a Point

Notes

Write your questions and thoughts here!

Formal Definition of Continuity:

For $f(x)$ to be continuous at $x = c$, the following three conditions must be met:

1.

2.

3.

1. State whether the function $f(x) = \begin{cases} x^2 - 2x + 1, & x < -1 \\ x + 2, & -1 \le x < 2 \\ 2^x, & x \ge 2 \end{cases}$ is continuous at the given x values. Justify your answers!

   a. $x = -1$

   b. $x = 2$

Identify the type of discontinuities (if any) and where they occur.

2. $f(x) = \begin{cases} 3x + 1, & x < 4 \\ \frac{x}{2} - 1, & x \ge 4 \end{cases}$

3. $g(x) = \begin{cases} x^2 + 2x - 1, & x < -1 \\ x - 1, & x > -1 \\ 5 & x = -1 \end{cases}$

4. Let h be the function defined by $h(x) = \begin{cases} 2 - x^2, & x \le -2 \\ 4x + k, & x > -2 \end{cases}$. What value of k would make h continuous?

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Practice

State whether the function is continuous at the given x values. Justify your answers!

1. $f(x) = \begin{cases} \frac{1}{x+4}, & x \le -1 \\ 3^x, & -1 < x < 2 \\ x^2 - 1, & x \ge 2 \end{cases}$

Continuous at $x = -1$?

Continuous at $x = 2$?

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2. $g(x) = \begin{cases} x - x^2, & x < 1 \\ \ln x, & x > 1 \\ x, & x = 1 \end{cases}$

Continuous at $x = 1$?

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3. $f(x) = \begin{cases} x^2 + 2x - 4, & x < -3 \\ 1^x, & -3 \le x \le 4 \\ 17 - x^2, & x > 4 \end{cases}$

Continuous at $x = -3$?

Continuous at $x = 4$?

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4. $f(x) = \begin{cases} \cos x, & x \le \frac{\pi}{2} \\ \tan x, & \frac{\pi}{2} < x < \pi \\ \sin x, & x \ge \pi \end{cases}$

Continuous at $x = \frac{\pi}{2}$?

Continuous at $x = \pi$?

For each function identify the type of each discontinuities and where they are is located.

5. $f(x) = \begin{cases} x^3 - 3x, & x < -2 \\ 3, & x = -2 \\ -\sqrt{x^2+2}, & -2 < x \le 4 \\ \ln x & x > 4 \end{cases}$

6. $f(x) = \begin{cases} 2x + 1, & x < -1 \\ 3, & x = -1 \\ -x^2 - 6x - 6, & -1 < x \le 1 \\ 3 & x > 1 \end{cases}$

7. $f(x) = \begin{cases} 2^x, & x < -2 \\ \frac{1}{4}, & x = -2 \\ 1 - \frac{1}{x^2}, & -2 < x < -1 \\ 5x - 1 & x \ge -1 \end{cases}$

8. $f(x) = \begin{cases} 1 - x^2, & x < 1 \\ -2, & x = 1 \\ \ln x, & 1 < x \le e \\ 4x & x > e \end{cases}$

For each function find the value *k* that makes the function continuous.

9. $f(x) = \begin{cases} 3 - x^2, & x \le 4 \\ x + k, & x > 4 \end{cases}$

10. $g(x) = \begin{cases} x^2 + k, & x \le -1 \\ 5x - 2, & x > -1 \end{cases}$

11. $h(x) = \begin{cases} (k + x)(3 + k), & x \le 2 \\ -\frac{x}{2} - 3k, & x > 2 \end{cases}$

12. $f(x) = \begin{cases} (k + 2x)(k - 4), & x \le 1 \\ kx + 2, & x > 1 \end{cases}$

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Test Prep

13. The function f has the properties indicated in the table below. Which of the following must be true?

b	$\lim_{n \to b^-} f(x)$	$\lim_{n \to b^+} f(x)$	$f(b)$
1	-1	3	3
2	5	5	8
3	1	1	1

(A) f is continuous at $x = 1$.

(B) f is continuous at $x = 2$.

(C) f is continuous at $x = 3$.

(D) None of the above.

CALCULATOR ACTIVE PROBLEM

14. Let f be the function $f(x) = \frac{x}{\ln x^2}$. Which of the following conditions explains why f is not continuous at $x = 1$.

(A) Both $\lim_{x \to 1} f(x)$ and $f(1)$ exist, but $\lim_{x \to 1} f(x) \neq f(1)$.

(B) $\lim_{x \to 1} f(x)$ exists, but $f(1)$ does not exist.

(C) Both $\lim_{x \to 1} f(x)$ and $f(1)$ exist, but $\lim_{x \to 1} f(x) = f(1)$.

(D) Neither $\lim_{x \to 1} f(x)$ nor $f(1)$ exists.