1.11 Defining Continuity at a Point

Calculus

Solutions

Practice

State whether the function is continuous at the given x values. Justify your answers!

1. $f(x) = \begin{cases} \frac{1}{x+4}, & x \le -1 \\ 3^x, & -1 < x < 2 \\ x^2 - 1, & x \ge 2 \end{cases}$

$\lim_{x\to -1^-} f(x) = \frac{1}{3}$ $\qquad$ $\lim_{x\to 2^-} f(x) = 9$

$\lim_{x\to -1^+} f(x) = \frac{1}{3}$ $\qquad$ $\lim_{x\to 2^+} f(x) = 8$

Continuous at $x = -1$?

Yes, because $f(-1) = \frac{1}{3}$ and $\lim_{x\to -1} f(x) = f(-1)$

Continuous at $x = 2$?

No, because $\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$

2. $g(x) = \begin{cases} x - x^2, & x < 1 \\ \ln x, & x > 1 \\ x, & x = 1 \end{cases}$

$\lim_{x\to 1^-} f(x) = 0$

$f(1) = 1$

$\lim_{x\to 1^+} f(x) = 0$

Continuous at $x = 1$?

No, because $\lim_{x\to 1} f(x) \neq f(1)$

3. $f(x) = \begin{cases} x^2 + 2x - 4, & x < -3 \\ 1^x, & -3 \le x \le 4 \\ 17 - x^2, & x > 4 \end{cases}$

$\lim_{x\to -3^-} h(x) = -1$ $\qquad$ $\lim_{x\to 4^-} f(x) = 1$

$\lim_{x\to -3^+} h(x) = 1$ $\qquad$ $\lim_{x\to 4^+} f(x) = 1$

Continuous at $x = -3$?

No, because $\lim_{x\to -3^-} f(x) \neq \lim_{x\to -3^+} f(x)$

Continuous at $x = 4$?

Yes, because $f(4) = 1$ and $\lim_{x\to 4} f(x) = f(4)$

4. $f(x) = \begin{cases} \cos x, & x \le \frac{\pi}{2} \\ \tan x, & \frac{\pi}{2} < x < \pi \\ \sin x, & x \ge \pi \end{cases}$

$\lim_{x\to \frac{\pi}{2}^-} f(x) = 0$ $\qquad$ $\lim_{x\to \pi^-} f(x) = 0$

$\lim_{x\to \frac{\pi}{2}^+} f(x) = \text{undefined}$ $\qquad$ $\lim_{x\to \pi^+} f(x) = 0$

Continuous at $x = \frac{\pi}{2}$?

No, because $\lim_{x\to \frac{\pi}{2}^-} f(x) \neq \lim_{x\to \frac{\pi}{2}^+} f(x)$

Continuous at $x = \pi$?

Yes, because $f(\pi) = 0$ and $\lim_{x\to \pi} f(x) = f(\pi)$

For each function identify the type of each discontinuities and where they are is located.

5. $f(x) = \begin{cases} x^3 - 3x, & x < -2 \\ 3, & x = -2 \\ -\sqrt{x^2+2}, & -2 < x \le 4 \\ \ln x & x > 4 \end{cases}$

$\lim_{x \to -2^-} f(x) = -2$

$\lim_{x \to -2^+} f(x) = -\sqrt{6}$

$\boxed{\text{Jump at } x = -2}$

$\lim_{x \to 4^-} f(x) = -\sqrt{18}$

$\lim_{x \to 4^+} f(x) = \ln 4$

$\boxed{\text{Jump at } x = 4}$

6. $f(x) = \begin{cases} 2x+1, & x < -1 \\ 3, & x = -1 \\ -x^2 - 6x - 6, & -1 < x \le 1 \\ 3 & x > 1 \end{cases}$

$\lim_{x \to -1^-} f(x) = -1$

$\lim_{x \to -1^+} f(x) = -1$

$\boxed{\text{Hole at } x = -1}$

$\lim_{x \to 1^-} f(x) = -13$

$\lim_{x \to 1^+} f(x) = 3$

$\boxed{\text{Jump at } x = 1}$

7. $f(x) = \begin{cases} 2^x, & x < -2 \\ \frac{1}{4}, & x = -2 \\ 1 - \frac{1}{x^2}, & -2 < x < -1 \\ 5x - 1 & x \ge -1 \end{cases}$

$\lim_{x \to -2^-} f(x) = \frac{1}{4}$

$\lim_{x \to -2^+} f(x) = \frac{3}{4}$

$\boxed{\text{Jump at } x = -2}$

$\lim_{x \to -1^-} f(x) = 0$

$\lim_{x \to -1^+} f(x) = -6$

$\boxed{\text{Jump at } x = -1}$

8. $f(x) = \begin{cases} 1 - x^2, & x < 1 \\ -2, & x = 1 \\ \ln x, & 1 < x \le e \\ 4x & x > e \end{cases}$

$\lim_{x \to 1^-} f(x) = 0$

$\lim_{x \to 1^+} f(x) = 0$

$\boxed{\text{Hole at } x = 1}$

$\lim_{x \to e^-} f(x) = 1$

$\lim_{x \to e^+} f(x) = 4e$

$\boxed{\text{Jump at } x = e}$

For each function find the value *k* that makes the function continuous.

9. $f(x) = \begin{cases} 3 - x^2, & x \le 4 \\ x + k, & x > 4 \end{cases}$

$3 - (4)^2 = (4) + k$

$3 - 16 = 4 + k$

$-13 = 4 + k$

$\boxed{-17 = k}$

10. $g(x) = \begin{cases} x^2 + k, & x \le -1 \\ 5x - 2, & x > -1 \end{cases}$

$(-1)^2 + k = 5(-1) - 2$

$1 + k = -7$

$\boxed{k = -8}$

11. $h(x) = \begin{cases} (k+x)(3+k), & x \le 2 \\ -\frac{x}{2} - 3k, & x > 2 \end{cases}$

$3k + k^2 + 3x + kx = -\frac{x}{2} - 3k$

$3k + k^2 + 3(2) + k(2) = -\frac{2}{2} - 3k$

$k^2 + 5k + 6 = -1 - 3k$

$k^2 + 8k + 7 = 0$

$(k+7)(k+1) = 0$

$\boxed{k = -7 \quad k = -1}$

12. $f(x) = \begin{cases} (k+2x)(k-4), & x \le 1 \\ kx + 2, & x > 1 \end{cases}$

$k^2 - 4k + 2kx - 8x = kx + 2$

$k^2 - 2k - 8 = k + 2$

$k^2 - 3k - 10 = 0$

$(k-5)(k+2) = 0$

$\boxed{k = 5, \ k = -2}$

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Test Prep

13. The function f has the properties indicated in the table below. Which of the following must be true?

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b	$\lim_{n\to b^-} f(x)$	$\lim_{n\to b^+} f(x)$	$f(b)$
1	-1	3	3
2	5	5	8
3	1	1	1

(A) f is continuous at $x = 1$.

(B) f is continuous at $x = 2$.

(C) f is continuous at $x = 3$.

(D) None of the above.

CALCULATOR ACTIVE PROBLEM

14. Let f be the function $f(x) = \frac{x}{\ln x^2}$. Which of the following conditions explains why f is not continuous at $x = 1$.

(A) Both $\lim_{x\to 1} f(x)$ and $f(1)$ exist, but $\lim_{x\to 1} f(x) \neq f(1)$.

(B) $\lim_{x\to 1} f(x)$ exists, but $f(1)$ does not exist.

(C) Both $\lim_{x\to 1} f(x)$ and $f(1)$ exist, but $\lim_{x\to 1} f(x) = f(1)$.

(D) Neither $\lim_{x\to 1} f(x)$ nor $f(1)$ exists.

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