

1.13 Removing Discontinuities

Calculus

1. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2-9}{x-3}$ when $x \neq 3$, then $f(3) =$

$$\frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}} \quad \lim_{x \rightarrow 3} f(x) = \boxed{6}$$

3. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2-5x+4}{x-1}$ when $x \neq 1$, then $f(1) =$

$$\frac{\cancel{(x-1)}(x-4)}{\cancel{x-1}} \quad \lim_{x \rightarrow 1} f(x) = \boxed{-3}$$

5. Let f be the function defined by

$$f(x) = \begin{cases} \frac{x^2-2x-15}{x-5}, & x \neq 5 \\ a, & x = 5 \end{cases}$$

For what value of a is f continuous at $x = 5$?

$$\frac{\cancel{(x-5)}(x+3)}{\cancel{x-5}} \quad 5+3 = \boxed{8}$$

7. Let f be the function defined by

$$f(x) = \begin{cases} \frac{x^2-8x}{x}, & x \neq 0 \\ c, & x = 0 \end{cases}$$

For what value of c is f continuous at $x = 0$?

$$\frac{\cancel{x}(x-8)}{\cancel{x}} \quad 0-8 = \boxed{-8}$$

9. Let f be the function defined by

$$f(x) = \begin{cases} \frac{x^2+5x+4}{b(x+1)}, & x \neq -1 \\ b, & x = -1 \end{cases}$$

For what value of b is f continuous at $x = -1$?

$$\frac{\cancel{(x+1)}(x+4)}{b\cancel{(x+1)}} \rightarrow \frac{3}{b} = b$$

$$\frac{-1+4}{b} = b$$

$$3 = b^2$$

$$\boxed{b = \pm\sqrt{3}}$$

Solutions

Practice

2. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2+8x-20}{x+10}$ when $x \neq -10$, then $f(-10) =$

$$\frac{\cancel{(x+10)}(x-2)}{\cancel{x+10}}, \quad \lim_{x \rightarrow -10} f(x) = \boxed{-12}$$

4. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2+14x+48}{x+8}$ when $x \neq -8$, then $f(-8) =$

$$\frac{\cancel{(x+8)}(x+6)}{\cancel{x+8}} \quad \lim_{x \rightarrow -8} f(x) = \boxed{-2}$$

6. Let f be the function defined by

$$f(x) = \begin{cases} \frac{x^2-16x+63}{x-7}, & x \neq 7 \\ b, & x = 7 \end{cases}$$

For what value of b is f continuous at $x = 7$?

$$\frac{\cancel{(x-7)}(x-9)}{\cancel{x-7}} \quad 7-9 = \boxed{-2}$$

8. Let f be the function defined by

$$f(x) = \begin{cases} \frac{x^2-8x+15}{x-3}, & x \neq 3 \\ a, & x = 3 \end{cases}$$

For what value of a is f continuous at $x = 3$?

$$\frac{\cancel{(x-3)}(x-5)}{\cancel{x-3}} \quad 3-5 = \boxed{-2}$$

10. Let f be the function defined by

$$f(x) = \begin{cases} \frac{x^2-49}{c(x+7)}, & x \neq -7 \\ c, & x = -7 \end{cases}$$

For what value of c is f continuous at $x = -7$?

$$\frac{\cancel{(x+7)}(x-7)}{c\cancel{(x+7)}} \rightarrow \frac{-14}{c} = c$$

$$\frac{-7-7}{c} = c$$

$$-14 = c^2$$

$$c = \pm\sqrt{-14}$$

No such value of c .

11. Let f be the function defined by

$$f(x) = \begin{cases} \frac{\sin(6x)}{5x}, & x \neq 0 \\ a, & x = 0 \end{cases}$$

For what value of a is f continuous at $x = 0$?

$$\lim_{x \rightarrow 0} f(x) = \frac{6}{5}$$

$$a = \frac{6}{5}$$

12. Let f be the function defined by

$$f(x) = \begin{cases} \frac{5 \sin(3x)}{4x}, & x \neq 0 \\ b, & x = 0 \end{cases}$$

For what value of b is f continuous at $x = 0$?

$$\lim_{x \rightarrow 0} f(x) = \frac{5 \cdot 3}{4} = \frac{15}{4}$$

$$b = \frac{15}{4}$$

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Test Prep

13. Let $y = \frac{x^2 + 4x - 21}{x^2 - 9}$. This function has a hole. What is the y -value of the hole?

A

$$\frac{(x+7)(x-3)}{(x-3)(x+3)} = \frac{x+7}{x+3}$$

$$\lim_{x \rightarrow 3} y = \frac{3+7}{3+3} = \frac{10}{6}$$

(A) $\frac{5}{3}$

(B) 3

(C) $-\frac{10}{3}$

(D) 0

(E) -3

14. For what value of k will the function $f(x) = \frac{x^2 - (k+2)x + 6}{x-k}$ have a point discontinuity at $x = k$?

Factor numerator $\rightarrow \frac{(x-k)(x-2)}{x-k}$

E

$$(-k)(-2) \text{ must equal } 6$$

(A) $k = -1$

(B) $k = 0$

(C) $k = 1$

(D) $k = 2$

(E) $k = 3$