1.14 Infinite Limits and Vertical Asymptotes

Calculus

Solutions

Practice

Identify the vertical asymptotes of each function.

1. $f(x) = \frac{\cancel{x-6}}{x^2-9x+18}$ $(\cancel{x-6})(x-3)$ $\boxed{x=3}$	2. $f(x) = \frac{2x^2-x-3}{3x^2+4x+1}$ $\frac{(2x-3)\cancel{(x+1)}}{(3x+1)\cancel{(x+1)}}$ $\boxed{x=-\frac{1}{3}}$	3. $f(x) = \frac{x^2-x-12}{x+7}$ $(x-4)(x+3)$ $\boxed{x=-7}$
4. $f(x) = \frac{3x^2-11x+10}{x-2}$ $\frac{(3x-5)\cancel{(x-2)}}{\cancel{(x-2)}}$ $\boxed{\text{no vertical asymptotes}}$	5. $f(x) = \frac{x^3+2x^2-24x}{x^2-x}$ $\frac{x(x+6)(x-4)}{x(x-1)}$ $\boxed{x=1}$	6. $f(x) = \frac{7x^2+4x-3}{7x-3}$ $\frac{\cancel{(7x-3)}(x+1)}{\cancel{7x-3}}$ $\boxed{\text{no vertical asymptotes}}$

7. $f(x) = \csc(2x)$ on the interval $[0, \pi]$

$f(x) = \frac{1}{\sin(2x)}$ $\quad \sin(2x) = 0$

$2x = 0 \quad 2x = \pi \quad 2x = 2\pi \quad 2x = 3\pi$

$\boxed{x = 0 \quad x = \frac{\pi}{2} \quad x = \pi}$ $\quad \cancel{x = \frac{3\pi}{2}}$

8. $f(x) = \sec\left(\frac{x}{2}\right)$ on the interval $[-\pi, \pi]$

$f(x) = \frac{1}{\cos(\frac{x}{2})}$ $\quad \cos(\frac{x}{2}) = 0$

$\frac{x}{2} = -\frac{\pi}{2} \quad \frac{x}{2} = \frac{\pi}{2} \quad \frac{x}{2} = \frac{3\pi}{2}$

$\boxed{x = -\pi \quad x = \pi}$ $\quad \cancel{x = 3\pi}$

Evaluate the limit.

9. $\lim_{x\to 1^+} \frac{x^2}{x-1}$

$\frac{(1.0001)^2}{1.0001 - 1}$

$\approx \frac{1}{0.0001}$

$\boxed{\infty}$

10. $\lim_{x\to -2^-} \frac{-3}{x+2}$

$\frac{-3}{-2.0001 + 2}$

$\frac{-3}{-0.0001}$

$\boxed{\infty}$

11. $\lim_{x\to 1^+} \frac{x-2}{x^2-3x+2} = \frac{\cancel{x-2}}{(\cancel{x-2})(x-1)}$

$\frac{1}{x-1} \rightarrow \frac{1}{1.0001 - 1}$

$\frac{1}{0.0001}$

$\boxed{\infty}$

12. $\lim_{x\to -2} \frac{x+3}{x^2+4x+4} = \frac{x+3}{(x+2)(x+2)}$

$x \to -2^- = \frac{-2.001 + 3}{(-2.001+2)(-2.001+2)} = \frac{0.999}{(-.001)(-.001)}$

∞

$x \to -2^+ = \frac{-1.999 + 3}{(0.001)(0.001)} = \frac{1.001}{\text{small}} = \infty$

$\boxed{\infty}$

13. $\lim_{x\to -1} \frac{x-1}{x^2-x-2}$ $\quad \frac{x-1}{(x-2)(x+1)}$

$x \to -1^- = \frac{-1.001 - 1}{(-1.001-2)(-1.001+1)} = \frac{-2.001}{\text{almost zero (positive)}}$

almost zero (but negative)

$-\infty$

$x \to -1^+ = \frac{-0.999 - 1}{(-0.999-2)(-0.999+1)} = \frac{-1.999}{\text{almost zero (negative)}}$

∞

$\boxed{\text{No Limit!}}$

14. $\lim_{x\to 3} -\frac{x^2}{3x-9}$

$x \to 3^- \approx -\frac{8.999}{8.999 - 9} \approx -\frac{8.999}{-0.001} = \infty$

$x \to 3^+ \approx -\frac{9.001}{9.001 - 9} \approx -\frac{9.001}{0.001} = -\infty$

$\boxed{\text{No Limit}}$

15. $\lim_{x\to -3} \frac{x-1}{x^2+6x+9} = \frac{(x-1)}{(x+3)(x+3)}$

$x \to -3^- \approx \frac{-3.001 - 1}{(-0.001)(-.001)} \approx \frac{-4.001}{0.000001} = -\infty$

$x \to -3^+ \approx \frac{-2.999 - 1}{(0.001)(0.001)} \approx \frac{-3.999}{0.000001} = -\infty$

$\boxed{-\infty}$

16. $\lim_{x \to 0^+} \frac{\cos x}{x} =$

$\frac{\cos(0.00001)}{0.00001} \to \frac{\text{almost } 1}{\text{almost zero}} \to \infty$

E

(A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞

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17. Consider the functions $f(x) = \frac{1}{x}, x \neq 0$, and $g(x) = x \sin\frac{1}{x}, x \neq 0$. Which of the following describes the behavior of f and g as $x \to 0$?

$\lim_{x \to 0} f(x) = \text{DNE}$

$\lim_{x \to 0} g(x) = (\text{almost zero})\underbrace{\sin(\infty)}$

$\lim_{x \to 0} g(x) = 0$

Sine is between −1 and 1.

(A) $\lim_{x \to 0} f(x) = 0$ and $\lim_{x \to 0} g(x) = 0$

(B) $\lim_{x \to 0} f(x)$ and $\lim_{x \to 0} g(x)$ do not exist.

(C) $\lim_{x \to 0} f(x) = 0$ and $\lim_{x \to 0} g(x)$ does not exist.

(D) $\lim_{x \to 0} f(x)$ does not exist and $\lim_{x \to 0} g(x) = 0$

(E) $\lim_{x \to 0} f(x) = \infty$ and $\lim_{x \to 0} g(x) = 0$

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18. The function h is defined by $h(x) = \left(\frac{x^2-x-20}{x+4}\right) \ln\left(\frac{x^2+10x+25}{x^2+5x}\right)$. At what values of x does the graph of h have a vertical asymptote?

$\frac{(x+4)(x-5)}{x+4} \ln\left[\frac{(x+5)(x+5)}{x(x+5)}\right]$

$(x-5)\ln\left(\frac{x+5}{x}\right)$

$\ln\left(\frac{x+5}{x}\right)^{x-5}$

properties of logarithms

$x=0$ is a V.A.

but if $\ln\left(\frac{x+5}{x}\right)$ has a negative exponent, then $\ln\left(\frac{x}{x+5}\right)$ has a V.A. at $x=-5$.

(A) $x = -5$ only

(B) $x = 0$ only

(C) $x = -5$ and $x = 0$ only

(D) $x = -5,\ x = 0$ and $x = -4$