Calculus

1.15 Limits at Infinity and Horizontal Asymptotes **Notes**

Write your questions and thoughts here!

Horizontal Asymptotes: (End-behavior)

What does the y-value approach as the x-value approaches negative infinity AND positive infinity? Does it approach a specific number, or is it growing without bound?

Basic Rules for Horizontal Asymptotes:

grows faster means $\frac{\text{not as big}}{\text{super duper BIG number!}} =$

If the numerator and denominator grow fast, then you have $\frac{\text{BIG number!}}{\text{BIG number!}} =$

If the grows faster than the denominator, then you have $\frac{\text{BIG number!}}{\text{not as big}} =$

First, you need to recognize which functions grow faster as x-values get larger and larger.

Rank Fastest to Slowest	$f(x)$	$x = 1$	$x = 10$	$x = 100$	$x = 1000$
	x^2	1	100	10,000	10^6
	x^3	1	1,000	10^6	10^9
	x^{10}	1	10^{10}	10^{20}	10^{30}
	2^x	2	1,024	1.26×10^{30}	1.07×10^{301}
	e^x	2.718	22,026	2.69×10^{43}	REALLY BIG
	4^x	4	1.05×10^6	1.6×10^{60}	SUPER-DUPER BIG
	$\ln x$	0	2.303	4.605	6.908

Find the horizontal asymptote(s) of each function.

4. $y = \frac{x^2+4}{3x-5}$

5. $y = \frac{x+4}{3x-5}$

6. $y = \frac{x+4}{3x^2-5}$

7. $f(x) = \frac{(x+5)(x-2)}{(4x-3)^2}$

8. $y = \frac{\sqrt{4x^2+x-2}}{3x-1}$

9. $y = \frac{\sqrt{4x^4+x-2}}{3x^2-1}$

Evaluate the limit.

10. $\lim_{x\to\infty} -4e^{\frac{1}{x}}$

11. $\lim_{x\to\infty} 5e^{-x}$

Write your questions and thoughts here!

Trig Functions' Horizontal Asymptotes:

Evaluate the limit.

12. $\lim_{x\to-\infty} \frac{\sin x}{x}$	13. $\lim_{x\to\infty} -3\cos\frac{1}{x}$	14. $\lim_{x\to\infty} \sin x$	15. $\lim_{x\to\infty} 5x\cos x$

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Practice

Identify all horizontal asymptotes of each function.

1. $f(x)=\frac{(2x-4)(3x+1)}{(2x-7)^2}$	2. $f(x)=\frac{(2x+5)(2-6x)}{(3x-2)^2}$	3. $f(x)=\frac{(5x-1)(x^2+4)}{(5x+1)^2}$

4. $f(x)=\frac{\sqrt{25x^4+2x}}{x^2}$	5. $f(x)=\frac{\sqrt{7x^6+3x^2+x}}{x^3+4x^2}$	6. $f(x)=\frac{\sqrt{9x^8-2x^3-6x}}{2x^4-10x}+3$	7. $f(x)=\frac{3x}{\sqrt{9x^2-1}}$

Evaluate each limit.

8. $\lim_{x\to\infty} \frac{-x+2}{x^2+2x+2}$	9. $\lim_{x\to\infty} \left(\sin\frac{1}{x}-\frac{6x^2+2x}{3x^2}\right)$	10. $\lim_{x\to\infty} \left(5\cos\frac{1}{x}\right)$	11. $\lim_{x\to\infty} \frac{x^7}{4^x}-5$
12. $\lim_{x\to\infty} 3^{-x}+2$	13. $\lim_{x\to\infty} -3x\cos x$	14. $\lim_{x\to\infty} 2x\sin x$	15. $\lim_{x\to\infty} \frac{9x^4+4x^3+3}{x^7+2x^4+2x^3}$
16. $\lim_{x\to-\infty} \frac{3x^2-5x+11}{x^2-2x}$	17. $\lim_{x\to\infty} \cos\left(\frac{2x-\pi x^2}{x^2}\right)$	18. $\lim_{x\to\infty} \left(\frac{\sin x}{x}-4\right)$	19. $\lim_{x\to\infty} \frac{-x^4-3x^2-8}{5x^4+7x+13}$
20. $\lim_{x\to\infty} \frac{x^3-7x^2+8}{x^2+7x-2}$	21. $\lim_{x\to\infty} x^2 2^{-x}$	22. $\lim_{x\to\infty} \frac{e^7}{9^x}$	23. $\lim_{x\to-\infty} \frac{3x^2-5x^7+6}{x^7-15x^4}$

24. $\lim_{x\to\infty} \frac{2x^4+3x^2+10}{5x^2+6x-1}$

25. $\lim_{x\to\infty} \left(\frac{\sin x}{x}+2\right)$

26. $\lim_{x\to\infty} \cos\left(\frac{x^5}{e^x}\right)+4$

27. $\lim_{x\to\infty} \frac{3x^6-5x^3+6}{x^3+x^8-2x^4}$

28. $\lim_{x\to\infty} \sin(2x)$

29. $\lim_{x\to\infty} \cos\left(\frac{\pi x^2+\frac{\sqrt{2}}{2}x}{5-2x^2}\right)$

30. $\lim_{x\to\infty} \cos\left(\frac{\frac{\sqrt{2}}{2}x-\pi x^2}{x^2-x^3+2}\right)$

1.15 Limits at Infinity and Horizontal Asymptotes

Test Prep

31. Which of the following functions grows the fastest?

(A) $a(u)=\left(\frac{1}{2}\right)^u$ (B) $b(u)=u^{100}+u^{99}$ (C) $c(u)=4^u$

(D) $d(u)=200e^u$ (E) $e(u)=3^u+u^3$

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32. Suppose that $g(x)=\sin^2 x\sqrt{x^6+4}$, and $\lim_{x\to\infty}\frac{g(x)}{f(x)}=0$. Which of the following functions could be f?

(A) x (B) x^2 (C) x^3 (D) x^4 (E) $\ln x$

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33. Which of the following statements are true for the function $f(x)=\frac{2x^3+3x+1}{2^x}$

I. $f(x)$ has a horizontal asymptote of $y=1$
II. $f(x)$ has a horizontal asymptote of $y=0$
III. $f(x)$ has a vertical asymptote of $x=0$

(A) I only (B) II only (C) III only (D) I and III only (E) II and III only

34. Which of the following functions has both a vertical and horizontal asymptote?

(A) $f(x) = \frac{1}{1+e^{-x}}$ (B) $f(x) = \tan x$ (C) $f(x) = \frac{x}{x^2+2}$

(D) $f(x) = \frac{x}{x^2-2}$ (E) $f(x) = \frac{x^2+2}{x}$

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35. The function $f(x) = \begin{cases} \frac{x^2+2x+3}{x^2-1}, & x \geq 0 \\ \frac{x}{e^x}, & x < 0 \end{cases}$ has which of the following asymptotes?

(A) $y = 0$ only. (B) $y = 1$ only. (C) $y = 1,\ x = 1$ only.

(D) $y = 1,\ x = \pm 1$ only. (E) $y = 0,\ y = 1,\ x = \pm 1$.

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36. If the function $f(x) = \frac{-ax^3+bx^2+cx+d}{e^{-x}-wx^3+w}$ has a horizontal asymptote of $y = 2$ and a vertical asymptote of $x = 0$, then $w - a =$

(A) -1 (B) 0 (C) 1 (D) ∞ (E) The limit does not exist.

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37. What are all horizontal asymptotes of the graph of $y = \frac{5+2^x}{1-2^x}$ in the xy-plane?

(A) $y = -1$ only (B) $y = 0$ only (C) $y = 5$ only

(D) $y = -1$ and $y = 0$ (E) $y = -1$ and $y = 5$