1.15 Limits at Infinity and Horizontal Asymptotes

Calculus

Solutions

Practice

Identify all horizontal asymptotes of each function.

1. $f(x)=\frac{(2x-4)(3x+1)}{(2x-7)^2}$

$\frac{6x^2+\cdots}{4x^2+\cdots}$

$y=\frac{3}{2}$

2. $f(x)=\frac{(2x+5)(2-6x)}{(3x-2)^2}$

$\frac{-12x^2+\cdots}{9x^2+\cdots}$

$y=-\frac{4}{3}$

3. $f(x)=\frac{(5x-1)(x^2+4)}{(5x+1)^2}$

$\frac{5x^3+\cdots}{25x^2+\cdots}$

No horizontal asymptote

4. $f(x)=\frac{\sqrt{25x^4+2x}}{x^2}$

Even is always positive

$\frac{5x^2+\cdots}{x^2}$

$y=5$

5. $f(x)=\frac{\sqrt{7x^6+3x^2+x}}{x^3+4x^2}$

$\frac{\sqrt{7}x^3+\cdots}{x^3}$

$y=\sqrt{7}$ and $y=-\sqrt{7}$

6. $f(x)=\frac{\sqrt{9x^8-2x^3-6x}}{2x^4-10x}+3$

Even is always positive

$\frac{3x^4+\cdots}{2x^4+\cdots}+3$

$y=\frac{9}{2}$

7. $f(x)=\frac{3x}{\sqrt{9x^2-1}}$

$\frac{3x}{3x+\cdots}$

$y=1$ and $y=-1$

Evaluate each limit.

8. $\lim_{x\to\infty}\frac{-x+2}{x^2+2x+2}$

$y=0$

9. $\lim_{x\to\infty}\left(\sin\frac{1}{x}-\frac{6x^2+2x}{3x^2}\right)$

$\sin(0)-\frac{6}{3}$

-2

10. $\lim_{x\to\infty}\left(5\cos\frac{1}{x}\right)$

$5\cos(0)$

5

11. $\lim_{x\to\infty}\frac{x^7}{4^x}-5$

$0-5$

-5

12. $\lim_{x\to\infty}3^{-x}+2$

$\frac{1}{3^x}+2$

2

13. $\lim_{x\to\infty}-3x\cos x$

DNE

Oscillating

14. $\lim_{x\to\infty}2x\sin x$

DNE

Oscillating

15. $\lim_{x\to\infty}\frac{9x^4+4x^3+3}{x^7+2x^4+2x^3}$

0

16. $\lim_{x\to-\infty}\frac{3x^2-5x+11}{x^2-2x}$

3

17. $\lim_{x\to\infty}\cos\left(\frac{2x-\pi x^2}{x^2}\right)$

$\cos(-\pi)$

-1

18. $\lim_{x\to\infty}\left(\frac{\sin x}{x}-4\right)$

$0-4$

-4

19. $\lim_{x\to\infty}\frac{-x^4-3x^2-8}{5x^4+7x+13}$

$-\frac{1}{5}$

20. $\lim_{x\to\infty}\frac{x^3-7x^2+8}{x^2+7x-2}$

∞

21. $\lim_{x\to\infty}x^2 2^{-x}$

$\frac{x^2}{2^x}$

0

22. $\lim_{x\to\infty}\frac{e^7}{9^x}$

0

23. $\lim_{x\to-\infty}\frac{3x^2-5x^7+6}{x^7-15x^4}$

$\frac{-5x^7+\cdots}{x^7+\cdots}$

-5

24. $\lim_{x\to\infty} \frac{2x^4+3x^2+10}{5x^2+6x-1}$

∞

25. $\lim_{x\to\infty} \left(\frac{\sin x}{x}+2\right)$

$0+2$

2

26. $\lim_{x\to\infty} \cos\left(\frac{x^5}{e^x}\right)+4$

$\cos(0)+4$

5

27. $\lim_{x\to\infty} \frac{3x^6-5x^3+6}{x^3+x^8-2x^4}$

0

28. $\lim_{x\to\infty} \sin(2x)$

DNE

29. $\lim_{x\to\infty} \cos\left(\frac{\pi x^2+\frac{\sqrt{2}}{2}x}{5-2x^2}\right)$

$\cos(-\pi/2)$

0

30. $\lim_{x\to\infty} \cos\left(\frac{\frac{\sqrt{2}}{2}x-\pi x^2}{x^2-x^3+2}\right)$

$\cos(0)$

1

1.15 Limits at Infinity and Horizontal Asymptotes

Test Prep

31. Which of the following functions grows the fastest?

(A) $a(u)=\left(\frac{1}{2}\right)^u$ (B) $b(u)=u^{100}+u^{99}$ (C) $c(u)=4^u$

(D) $d(u)=200e^u$ (E) $e(u)=3^u+u^3$

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32. Suppose that $g(x)=\sin^2 x\sqrt{x^6+4}$, and $\lim_{x\to\infty}\frac{g(x)}{f(x)}=0$. Which of the following functions could be f?

$f(x) > g(x)$ and $g(x) \approx x^3 + \cdots$

(A) x (B) x^2 (C) x^3 (D) x^4 (E) $\ln x$

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33. Which of the following statements are true for the function $f(x)=\frac{2x^3+3x+1}{2^x}$

I. $f(x)$ has a horizontal asymptote of $y=1$
II. $f(x)$ has a horizontal asymptote of $y=0$
III. $f(x)$ has a vertical asymptote of $x=0$

$\neq 0 \rightarrow$ no vert. asym.

$x\to\infty$ gives $y=0$ as H.A.

$x\to-\infty$ no H.A.

(A) I only (B) II only (C) III only (D) I and III only (E) II and III only

34. Which of the following functions has both a vertical and horizontal asymptote?

$1+e^{-x}=0$

$\ln(e^{-x})=\ln(-1)$

D

(A) $f(x)=\frac{1}{1+e^{-x}}$ — no V.A.

(B) $f(x)=\tan x$ — no H.A.

(C) $f(x)=\frac{x}{x^2+2}$ → no V.A.

(D) $f(x)=\frac{x}{x^2-2}$

(E) $f(x)=\frac{x^2+2}{x}$ — no H.A.

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35. The function $f(x)=\begin{cases}\frac{x^2+2x+3}{x^2-1}, & x\geq 0\\ \frac{x}{e^x}, & x<0\end{cases}$ has which of the following asymptotes?

$\frac{x^2+2x+3}{x^2-1}$ → $y=1$, $x=1$, ~~$x=-1$~~ not in domain of $x\geq 0$.

$\frac{x}{e^x}$ → $\frac{-\infty}{\frac{1}{e^{\infty}}}$ → $-\frac{\infty}{0}$ → no H.A

C

(A) $y=0$ only.

(B) $y=1$ only.

(C) $y=1,\ x=1$ only.

(D) $y=1,\ x=\pm 1$ only.

(E) $y=0,\ y=1,\ x=\pm 1$.

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36. If the function $f(x)=\frac{-ax^3+bx^2+cx+d}{e^{-x}-wx^3+w}$ has a horizontal asymptote of $y=2$ and a vertical asymptote of $x=0$, then $w-a=$

This second → horizontal asymptote

This first → vertical asymptote

as $x\to\infty$ $\frac{-ax^3+\dots}{\frac{1}{e^{\infty}}-wx^3+\dots}=2$

$-\frac{a}{-w}=2$

$a=-2$

$e^{-0}-w(0)^3+w=0$

$1+w=0$

$w=-1$

C

(A) -1

(B) 0

(C) 1

(D) ∞

(E) The limit does not exist.

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37. What are all horizontal asymptotes of the graph of $y=\frac{5+2^x}{1-2^x}$ in the xy-plane?

as $x\to\infty$ $\frac{2^{\infty}}{-2^{\infty}}=-1$

as $x\to-\infty$ $\frac{5+\frac{1}{2^{\infty}}}{1-\frac{1}{2^{\infty}}}=\frac{5+0}{1-0}=5$

E

(A) $y=-1$ only

(B) $y=0$ only

(C) $y=5$ only

(D) $y=-1$ and $y=0$

(E) $y=-1$ and $y=5$