1.16 Intermediate Value Theorem (IVT)

Calculus

Practice

Below is a table of values for a continuous function f.

x	-5	1	3	8	14
$f(x)$	7	40	21	75	-100

1. On the interval $-5 \le x \le 1$, must there be a value of x for which $f(x) = 30$? Explain.

   $f(-5) = 7$ $f(1) = 40$

   According to the IVT, there is a value c such that $f(c) = 30$ and $-5 \le c \le 1$.

2. On the interval $3 \le x \le 8$, ***could*** there be a value of x for which $f(x) = 100$? Explain.

   Yes, there could be, but the IVT does not guarantee it.

3. On the interval $-5 \le x \le 14$ what is the minimum number of zeros? $\boxed{1}$

4. For $1 \le x \le 14$, what is the fewest possible number of times $f(x) = 20$? $\boxed{1}$

5. For $1 \le x \le 8$, what is the fewest possible number of times $f(x) = 7$? $\boxed{0}$

Below is a table of values for a continuous function h.

x	-7	-2	1	4	11
$h(x)$	2	-5	6	-1	10

6. For $-7 \le x \le 1$, what is the fewest possible number of times $f(x) = 3$? $\boxed{1}$

7. On the interval $4 \le x \le 11$, must there be a value of x for which $f(x) = -2$? Explain.

   $f(4) = -1$ $f(11) = 10$

   No, the IVT cannot guarantee $f(x) = -2$ because the smallest y-value is -1.

8. For $-2 \le x \le 4$, what is the fewest possible number of times $f(x) = 2$? $\boxed{2}$

9. On the interval $1 \le x \le 11$, ***could*** there be a value of x for which $f(x) = -2$? Explain.

   Yes, there could be, but the IVT does not guarantee it.

10. On the interval $-7 \le x \le 11$ what is the minimum number of zeros? $\boxed{4}$

Below is a table of values for a continuous function g.

x	0	2	15	32	50
$g(x)$	-1	10	17	-10	8

11. On the interval $2 \le x \le 15$, must there be a value of x for which $g(x) = -3$? Explain.

$g(2) = 10$ $g(15) = 17$

$g(x)$ might be -3 at some point, but the IVT does not guarantee it because the smallest guaranteed value is 10.

12. On the interval $15 \le x \le 32$, must there be a value of x for which $g(x) = 11$? Explain.

$g(15) = 17$ $g(32) = -10$

According to the IVT, there is a value c such that $f(c) = 11$ and $15 \le c \le 32$.

13. What is the minimum number of zeros g must have on the interval $15 \le x \le 50$? $\boxed{2}$

14. What is the minimum number of zeros g must have on the interval $0 \le x \le 50$? $\boxed{3}$

15. For $15 \le x \le 50$, what is the fewest possible number of times $g(x) = 1$? $\boxed{2}$

Use the Intermediate Value Theorem to answer each problem.

16. If $f(x) = 3 - x^2$, will $f(x) = 0$ on the interval $[-2, 1]$? Explain.

$f(x)$ is a continuous polynomial function.

$f(-2) = -1$ and $f(1) = 2$

According to the IVT, there is a value c such that $f(c) = 0$ and $-2 \le c \le 1$.

17. If $g(x) = \frac{1}{x}$, will $g(x) = -1$ on the interval $[2, 5]$? Explain.

$g(x)$ is discontinuous at $x = 0$, but is continuous on the interval $[2, 5]$.

$g(2) = \frac{1}{2}$ and $g(5) = \frac{1}{5}$

There is no guarantee that $g(x) = -1$ because the smallest value is $\frac{1}{5}$.

18. **Calculator active.** If $h(x) = \ln(2x + 1)$, will $h(x) = 3$ on the interval $[2, 20]$? Explain.

$h(x)$ is continuous on $x > -\frac{1}{2}$

$h(2) \approx 1.609$ and $h(20) \approx 3.7135$

According to the IVT, there is a value c such that $h(c) = 3$ and $2 \le c \le 20$.

19. If $f(t) = 3t^2 - 10t + 2$, will $f(x) = 1$ on the interval $[-1, 3]$? Explain.

$f(t)$ is a continuous polynomial function.

$f(-1) = 15$ and $f(3) = -1$

According to the IVT, there is a value c such that $f(c) = 1$ and $-1 \le c \le 3$.

1.16 Intermediate Value Theorem (IVT) — Test Prep

20. Let f be a continuous function such that $f(1) = 7$ and $f(7) = 1$. Let g be the function given by $g(x) = f(x) - x$. Explain why there must be a value c for $1 < c < 7$ such that $g(c) = 0$.

$g(1) = f(1) - 1 = 7 - 1 = 6$

$g(7) = f(7) - 7 = 1 - 7 = -6$

By the IVT, there must be a value c on $1 < c < 7$ where $g(c) = 0$.

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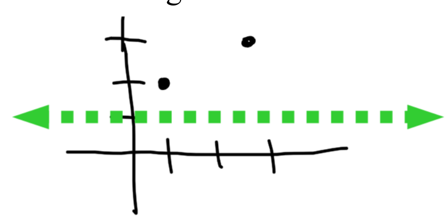
21. The function f is continuous on the closed interval $[1, 3]$ and has values that are given in the table below.

x	1	2	3
$f(x)$	2	k	3



A

The equation $g(x) = 1$ must have at least two intersections with f in the interval $[1, 3]$ if $k =$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

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22. Suppose f is continuous on the closed interval $[0,4]$ and suppose $f(0) = 1, f(1) = 2, f(2) = 0, f(3) = -3, f(4) = 3$. Which of the following statements about the zeros of f on $[0,4]$ is always true?

B

False. Could be more

could be exactly two

(A) f has exactly one zero on $[0, 4]$.

(B) f has more than one zero on $[0, 4]$.

(C) f has more than two zeros on $[0, 4]$.

(D) f has exactly two zeros on $[0, 4]$.

Could be more.

(E) None of the statements above is true.