1.5 Algebraic Properties of Limits

Calculus

Solutions

Practice

Use the table for each problem to find the given limits.

1.

$\lim_{x\to3} f(x) = 4$	$\lim_{x\to-3} f(x) = 2$	$\lim_{x\to3} g(x) = 1$	$\lim_{x\to-3} g(x) = 5$

a. $\lim_{x\to3}\left(2f(x) + g(-x)\right)$

$2(4) + 5$

$\boxed{13}$

b. $\lim_{x\to-3}\left(\frac{g(x)}{f(-x)}\right) = \boxed{\frac{5}{4}}$

2.

$\lim_{x\to2} f(x) = -1$	$\lim_{x\to1} f(x) = 6$	$\lim_{x\to4} f(x) = 2$	$\lim_{x\to-2} f(x) = -3$

a. $\lim_{x\to2}\left(f(2x) - f\left(\frac{x}{2}\right)\right)$

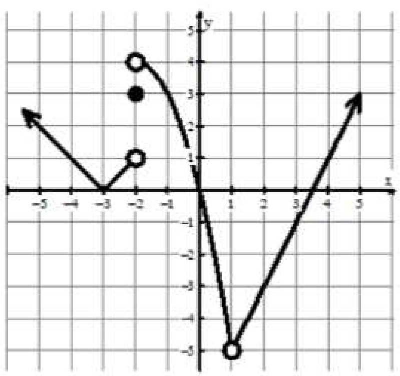
$2 - 6$

$\boxed{-4}$

b. $\lim_{x\to2}\left(\frac{f\left(\frac{x}{2}\right)}{f(-x)}\right) = \frac{6}{-3} = \boxed{-2}$

Use the graph for each problem to find the given limits.

3.

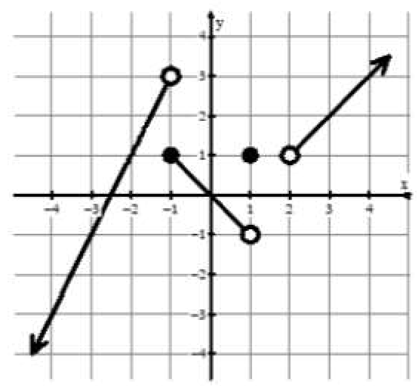


Graph of f

a. $\lim_{x\to3} f(f(x)) = \lim_{x\to-1} f(x) = \boxed{3}$

b. $\lim_{x\to1} f(f(x)) = \lim_{x\to-5} f(x) = \boxed{2}$

4.



Graph of f

a. $\lim_{x\to-2} f(f(x)) = \lim_{x\to1} f(x) = \boxed{\text{Does not exist}}$

b. $\lim_{x\to4} f(f(x)) = \lim_{x\to3} f(x) = \boxed{2}$

Use the table for each problem to find the given limits.

5.

$f(1) = 4$	$\lim_{x\to1} f(x) = -1$
$g(1) = -2$	$\lim_{x\to1} g(x) = 3$
$h(1) = -3$	$\lim_{x\to1} h(x) = 6$

a. $\lim_{x\to1}\left(\left(f(x)\right)^2 - h(x)\right) - g(1)$

$(-1)^2 - 6 \quad -(-2)$

$-5 + 2$

$\boxed{-3}$

b. $f(1) + \lim_{x\to1}\left(-g(x)\right)$

$4 + \left(-(3)\right)$

$4 - 3$

$\boxed{1}$

6.

$f(-2) = 7$	$\lim_{x\to-2} f(x) = 2$
$g(-2) = 1$	$\lim_{x\to-2} g(x) = -1$
$h(-2) = -4$	$\lim_{x\to-2} h(x) = -3$

a. $\lim_{x\to-2}\left(h(x)(2f(x))\right) + h(-2)$

$((-3)(2\cdot 2)) + (-4)$

$-12 - 4 = \boxed{-16}$

b. $f(-2)\lim_{x\to-2}(g(x) - h(x))$

$7\cdot(-1 - -3)$

$7(2) = \boxed{14}$

Use the piecewise functions to find the given values.

7. $g(x) = \begin{cases} \sqrt{5-x}, & x < -4 \\ x^2 - 5, & -4 \le x < 2 \\ x - 3, & x \ge 2 \end{cases}$

a. $\lim_{x\to2^-} g(x) =$ $(2^2) - 5 = \boxed{-1}$

b. $\lim_{x\to-4^+} g(x) =$ $(-4)^2 - 5 = \boxed{11}$

c. $g(2) =$ $(2) - 3 = \boxed{-1}$

d. $\lim_{x\to-4^-} g(x) =$ $\sqrt{5-(-4)} = \sqrt{9} = \boxed{3}$

e. $\lim_{x\to2^+} g(x) =$ $(2) - 3 = \boxed{-1}$

f. $\lim_{x\to2} g(x) =$ $\boxed{-1}$

g. $\lim_{x\to-4} g(x) =$ $\boxed{\text{Does not exist}}$

h. $g(-4) =$ $(-4)^2 - 5 = \boxed{11}$

8. $h(x) = \begin{cases} -|x|, & x \le -5 \\ 20 - x^2, & -5 < x \le 3 \\ 4x - 1, & x > 3 \end{cases}$

a. $\lim_{x\to-5^+} h(x) =$ $20 - (5)^2 = \boxed{-5}$

b. $\lim_{x\to-5} h(x) =$ $\boxed{-5}$

c. $h(3) =$ $20 - (3)^2 = \boxed{11}$

d. $\lim_{x\to-5^-} h(x) =$ $-|-5| = \boxed{-5}$

e. $\lim_{x\to3^+} h(x) =$ $4(3) - 1 = \boxed{11}$

f. $\lim_{x\to3} h(x) =$ $\boxed{11}$

g. $h(-5) =$ $-|-5| = \boxed{-5}$

h. $\lim_{x\to3^-} h(x) =$ $20 - (3)^2 = \boxed{11}$

9. $w(\theta) = \begin{cases} \sin\theta, & \theta \le \pi \\ \cos\theta, & \pi < \theta < 2\pi \\ \tan\theta, & \theta > 2\pi \end{cases}$

a. $\lim_{x\to\pi^-} w(\theta) =$ $\sin(\pi) = \boxed{0}$

b. $w(\pi) =$ $\sin(\pi) = \boxed{0}$

c. $\lim_{x\to\pi^+} w(\theta) =$ $\cos(\pi) = \boxed{-1}$

d. $\lim_{x\to2\pi^-} w(\theta) =$ $\cos(2\pi) = \boxed{1}$

e. $\lim_{x\to\pi} w(\theta) =$ $\boxed{\text{Does not exist}}$

f. $\lim_{x\to2\pi^+} w(\theta) =$ $\tan(2\pi) = \boxed{0}$

g. $\lim_{x\to2\pi} w(\theta) =$ $\boxed{\text{Does not exist}}$

h. $w(2\pi) =$ $\boxed{\text{Does not exist}}$

10. $f(x) = \begin{cases} \frac{1}{x+6}, & x < -2 \\ 2^x, & -2 \le x < 0 \\ x^2 - 4, & x \ge 0 \end{cases}$

a. $\lim_{x\to-2} f(x) =$ $\boxed{1/4}$

b. $\lim_{x\to-2^-} f(x) =$ $\frac{1}{-2+6} = \boxed{1/4}$

c. $\lim_{x\to-2^+} f(x) =$ $2^{-2} = \frac{1}{2^2} = \boxed{1/4}$

d. $\lim_{x\to0} f(x) =$ $\boxed{\text{Does not exist}}$

e. $\lim_{x\to0^-} f(x) =$ $2^0 = \boxed{1}$

f. $\lim_{x\to0^+} f(x) =$ $0 - 4 = \boxed{-4}$

g. $f(-2) =$ $2^{-2} = \frac{1}{2^2} = \boxed{1/4}$

h. $f(0) =$ $0^2 - 4 = \boxed{-4}$

11. If f is a continuous function such that $f(3) = 7$, which of the following statements must be true?

(A) $\lim_{x\to 3} f(3x) = 9$ (B) $\lim_{x\to 3} f(2x) = 14$ (C) $\lim_{x\to 3} \frac{f(x)-f(3)}{x-3} = 7$

(D) $\lim_{x\to 3} f(x^2) = 49$ (E) $\lim_{x\to 3} \left(f(x)\right)^2 = 49$ E

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12. If $f(x) = \begin{cases} \ln 3x, & 0 < x \leq 3 \\ x \ln 3, & 3 < x \leq 4 \end{cases}$, then $\lim_{x\to 3} f(x)$ is E

$\ln 3^x$ $\ln 27 = \ln 9$?

(A) $\ln 9$ (B) $\ln 27$ (C) $3 \ln 3$ (D) $3 + \ln 3$ (E) nonexistent

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13. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \leq 2 \\ x^2 \ln 2 & \text{for } 2 < x \leq 4, \end{cases}$ then $\lim_{x\to 2} f(x)$ is E

$\ln 2 = 2^2 \ln 2$

$\ln 2 = 4 \cdot \ln 2$ False!

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent