## 1.8 The Squeeze Theorem

Calculus Name: ______________________________ **CA #1**

1. $g(x) = -3x^2 + x + 1$ and $h(x) = x - 2$. If $f$ is a function that satisfies $g(x) \le f(x) \le h(x)$ for all $x$, what is $\lim_{x \to -1} f(x)$?

2. $g(x) = -x^2 + x$ and $h(x) = x^2 - x$. If $f$ is a function that satisfies $g(x) \le f(x) \le h(x)$ for $1 \le x \le 3$, what is $\lim_{x \to 2} f(x)$?

3. $g(x) = -\frac{1}{2}x^2 + x - \frac{9}{2}$ and $h(x) = \cos(\pi(x+2)) - 3$. If $f$ is a function that satisfies $g(x) \le f(x) \le h(x)$ for all $x$, what is $\lim_{x \to 1} f(x)$?

4. $g(x) = \sin\left(\frac{\pi}{2}(x-1)\right) + 2$ and $h(x) = 2x^2 + 1$. If $f$ is a function that satisfies $g(x) \le f(x) \le h(x)$ for all $x$, what is $\lim_{x \to 0} f(x)$?

5. Let $f$ and $g$ be the functions defined by $f(x) = \frac{\sin x}{4x}$ and $g(x) = x^5 \cos\left(\frac{1}{x^3}\right)$ for $x \neq 0$. The following inequalities are true for $x \neq 0$. State whether each inequality can be used with the squeeze theorem to find the limit of the function as $x$ approaches 0?

   a. $\frac{1}{4} \le f(x) \le \frac{1}{2}$

   b. $-x^5 \le g(x) \le x^5$

   c. $-\frac{1}{x^3} \le g(x) \le \frac{1}{x^3}$

6. Let $f$ and $g$ be the functions defined by $f(x) = \frac{6 - 6\cos x}{x^2}$ and $g(x) = x^3 \cos\left(\frac{1}{x}\right)$ for $x \neq 0$. The following inequalities are true for $x \neq 0$. State whether each inequality can be used with the squeeze theorem to find the limit of the function as $x$ approaches 0?

   a. $3 - x^2 \le f(x) \le 3$

   b. $-x^4 \le f(x) \le 1 + x^2$

   c. $-x^3 \le g(x) \le x^3$

Answers to 1.8 CA #1

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|---|---|---|---|---|
| 1. $-3$ | 2. Cannot be determined. | 3. $-4$ | 4. 1 | 5a. No. The upper and lower limits are not the same. |
| 5b. Yes. Both equal 0. | 5c. No. The outside functions are not bounded. | 6a. Yes. Both equal 3. | 6b. No. The upper and lower limits are not the same. | 6c. Yes. Both equal 0. |