1.8 The Squeeze Theorem

Calculus Name: ______________________________ **CA #2**

1. $g(x) = -x^2 + 4x - 1$ and $h(x) = 5x - 3$. If f is a function that satisfies $g(x) \leq f(x) \leq h(x)$ for all x, what is $\lim_{x \to 1} f(x)$?

2. $g(x) = \cos(\pi(x-1)) + 1$ and $h(x) = \frac{1}{2}x^2 + x + \frac{5}{2}$. If f is a function that satisfies $g(x) \leq f(x) \leq h(x)$ for $-2 \leq x \leq 0$, what is $\lim_{x \to -1} f(x)$?

3. $g(x) \sin(\pi(x+1)) - 1$ and $h(x) = \sin\left(\frac{\pi}{2}x\right) + 2$. If f is a function that satisfies $g(x) \leq f(x) \leq h(x)$ for all x, what is $\lim_{x \to 1} f(x)$?

4. $g(x) = -x^2 - 2x + 1$ and $h(x) = 7 - x^2$. If f is a function that satisfies $g(x) \leq f(x) \leq h(x)$ for all x, what is $\lim_{x \to -3} f(x)$?

5. Let f and g be the functions defined by $f(x) = \frac{\sin x}{7x}$ and $g(x) = x^4 \cos\left(\frac{1}{x}\right)$ for $x \neq 0$. The following inequalities are true for $x \neq 0$. State whether each inequality can be used with the squeeze theorem to find the limit of the function as x approaches 0?

   a. $\frac{1}{7} \leq f(x) \leq x^2 + \frac{1}{7}$

   b. $-\frac{1}{7} \leq f(x) \leq \frac{1}{7}$

   c. $-x^4 \leq g(x) \leq x^4$

6. Let f and g be the functions defined by $f(x) = \frac{4 - 4\cos x}{x^2}$ and $g(x) = x^2 \sin\left(\frac{1}{x}\right)$ for $x \neq 0$. The following inequalities are true for $x \neq 0$. State whether each inequality can be used with the squeeze theorem to find the limit of the function as x approaches 0?

   a. $2(1 - x^2) \leq f(x) \leq 2$

   b. $-x^2 \leq g(x) \leq x^2$

Answers to 1.8 CA #2

1. 2	2. 2	3. Cannot be determined.	4. −2	5a. Yes. Both equal 0.
5b. No. The upper and lower limits are not the same.	5c. Yes. Both equal 0.	6a. Yes. Both equal 2.	6b. Yes. Both equal 0.	