1.9 Connecting Multiple Representations of Limits

Calculus

Solutions

Practice

Evaluate each limit.

1. $\lim_{x \to 7^-} \frac{|x-7|}{x-7}$

$\frac{|6.999-7|}{6.999-7} = \frac{|-0.001|}{-0.001}$

$\boxed{-1}$

2. $\lim_{x \to 3^+} \frac{x-3}{|x-3|}$

$\frac{3.001-3}{|3.001-3|} = \frac{|0.001|}{0.001}$

$\boxed{1}$

3. $\lim_{x \to -1^+} \frac{|x+1|}{x+1}$

$\frac{|-0.999+1|}{-0.999+1} = \frac{|0.001|}{0.001}$

$\boxed{1}$

4. $\lim_{x \to 8^+} \frac{|x-8|}{x-8}$

$\frac{|8.001-8|}{8.001-8} = \frac{|0.001|}{0.001}$

$\boxed{1}$

5. $\lim_{x \to 2^-} \frac{|x-2|}{x-2}$

$\frac{|1.999-2|}{1.999-2} = \frac{|-0.001|}{-0.001}$

$\boxed{-1}$

6. $\lim_{x \to -9^-} \frac{x+9}{|x+9|}$

$\frac{-9.001+9}{|-9.001+9|} = \frac{-0.001}{|-0.001|}$

$\boxed{-1}$

7. Let f be a function where $\lim_{x \to 5} f(x) = \frac{1}{4}$. Which of the following could represent the function f?

a.

$$f(x) = \begin{cases} \frac{x-5}{x^2-6x+5}, & x \neq 5 \\ 4, & x = 5 \end{cases}$$

$\frac{\cancel{x-5}}{(\cancel{x-5})(x-1)}$

$\frac{1}{x-1}$

$\frac{1}{5-1} = \frac{1}{4}$

$\boxed{\text{Yes}}$

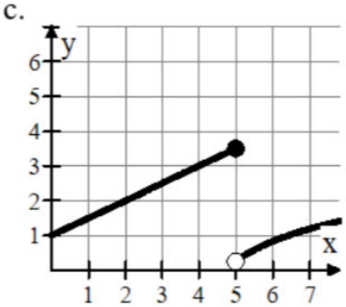
b.

x	4.8	4.9	4.999	5	5.001	5.1	5.2
$f(x)$	0	0.2	0.249	4	0.251	0.3	0.5

$\xrightarrow{0.25}$ $\xleftarrow{0.25}$

$\boxed{\text{Yes}}$

c.



$\boxed{\text{No}}$

8. Let g be a function where $\lim_{x \to 3} g(x) = 6$. Which of the following could represent the function g?

a.

$$g(x) = \begin{cases} \frac{x^2-x-6}{x-3}, & x \neq 3 \\ 6, & x = 3 \end{cases}$$

$\frac{(\cancel{x-3})(x+2)}{\cancel{x-3}}$

$x+2$

$3+2=5$ $\boxed{\text{No}}$

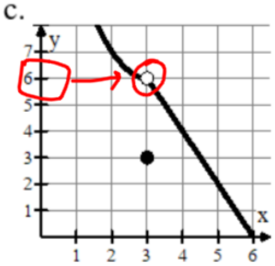
b.

x	2.8	2.9	2.999	3	3.001	3.1	3.2
$g(x)$	6.2	6.01	6.001	1	4.999	4.9	4.8

$\xrightarrow{6}$ $\xleftarrow{5}$

$\boxed{\text{No}}$

c.



$\boxed{\text{Yes}}$

9. If h is a piecewise linear function such that $\lim_{x \to 4} h(x)$ does not exist, which of the following could be representative of the function h?

a.

$$h(x) = \begin{cases} \frac{1}{2}x + 3, & x < 4 \\ 13 - 2x, & x > 4 \end{cases}$$

$\frac{1}{2}(4) + 3 = 5$

$13 - 2(4) = 5$

No, the Limit exists

b.

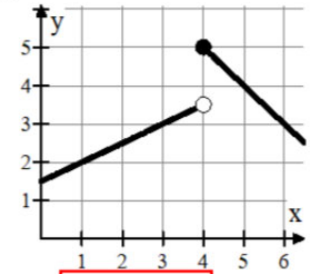
x	1	2	3	4	5	6	7
$h(x)$	-4	-1	2	1	$\frac{16}{3}$	$\frac{17}{3}$	$\frac{18}{3}$

+3 +3 $-\frac{1}{3}$ $-\frac{1}{3}$

$\xrightarrow{\quad} 5 \qquad \xleftarrow{\quad} \frac{15}{3}$

No, the Limit exists.

c.



Yes

10. If $f(x) = \begin{cases} \frac{(x-3)^2(x^2+1)}{|x-3|} & \text{for } x \neq 3 \\ 2 & \text{for } x = 3 \end{cases}$, then $\lim_{x \to 3} f(x)$ is $\frac{(\text{almost zero})^2(3^2+1)}{|(\text{almost zero})|} = \frac{\text{Super Small}}{\text{Small}} = 0$

Both positive

$\boxed{\text{(A) } 0}$ (B) 2 (C) 10 (D) Nonexistent