Calculus

10.10 Alternating Series Error Bound

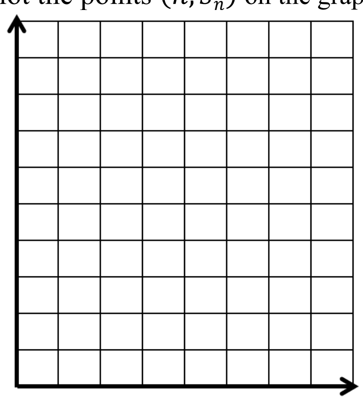
Notes

Write your questions and thoughts here!

Use the alternating harmonic series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ to fill in the table below.

n		1	2	3	4	5	6	7	8
a_n	fractions								
	decimals								
S_n	fractions								
	decimals								

Plot the points (n, S_n) on the graph.



Error: $\lvert S - S_n \rvert$								

Alternating Series Error Bound

If you have an alternating series that converges, we can approximate the sum of the series!

$$= \qquad \leq$$

S: Sum of the series

S_n: Partial sum

R_n: Remainder (or error)

$R_n = S - S_n$

a_{n+1} = next term (Error Bound)

Write your questions and thoughts here!

1. Determine the number of terms required to approximate the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ with an error less than 10^{-3}.

2. If the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{5n+2}$ is approximated by the partial sum with 10 terms, what is the alternating series error bound?

3. **Calculator active.** Approximate an interval of the sum of the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{4}{n^2}$ using the Alternating Series Error Bound for the first 5 terms.

4. Let $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots$. Show that $1 - \frac{1}{3!}$ approximates $f(1)$ with an error less than 0.01.

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Practice

A calculator may be used on all problems in this practice. For 1-2, approximate an interval of the sum of the alternating series using the Alternating Series Error Bound for the first 6 terms.

1. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}n}{3^n}$

2. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}4}{\ln(n+2)}$

3. Determine the number of terms needed to approximate the sum of the series $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}$ with an error less than 10^{-3}.

4. The series $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt{n}}$ converges to S. Using the alternating series bound, what is the least number of terms that must be summed to guarantee a partial sum that is within 0.05 of S?

(A) 20 (B) 55 (C) 399 (D) 400

5. If the infinite series $S = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{4}{n}$ is approximated by $P_k = \sum_{n=1}^{k} (-1)^{n+1} \frac{4}{n}$, what is the least value of k for which the alternating series error bound guarantees that $|S - P_k| < \frac{7}{100}$?

(A) 55 (B) 56 (C) 57 (D) 60

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6. If the series $S = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^3}$ is approximated by the partial sum $S_k = \sum_{n=1}^{k} (-1)^{n+1} \frac{1}{n^3}$, what is the least value of k for which the alternating series error bound guarantees that $|S - S_k| \leq \frac{7}{10000}$?

(A) 10 (B) 11 (C) 12 (D) 13

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7. The series $\sum_{k=1}^{\infty} (-1)^{k+1} a_k$ converges by the alternating series test. If $S_n = \sum_{k=1}^{n} (-1)^{k+1} a_k$ is the nth partial sum of the series, which of the following statements must be true?

(A) $\lim_{n\to\infty} S_n = 0$ (B) $\lim_{n\to\infty} a_n = S$ (C) $|S - S_{20}| \leq a_{26}$ (D) $|S - S_{25}| \leq a_{26}$

8. If the series $\sum_{n=1}^{\infty}(-1)^n \frac{1}{5n+1}$ is approximated by the partials sum with 15 terms, what is the alternating series error bound?

(A) $\frac{1}{15}$ (B) $\frac{1}{16}$ (C) $\frac{1}{76}$ (D) $\frac{1}{81}$

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9. The function f is defined by the power series $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x}{(2n+1)!}$ for all real numbers x. Show that $1 - \frac{1}{3!} + \frac{1}{5!}$ approximates $f(1)$ with an error less than $\frac{1}{4000}$.

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Test Prep

10. **Calculator active!** Let $f(x) = \sum_{n=1}^{\infty} \frac{x^n n^n}{n!}$ for all x for which the series converges.

a. Use the first three terms of the series to approximate $f\left(-\frac{1}{3}\right)$.

b. How far off is this estimate from the value of $f\left(-\frac{1}{3}\right)$? Justify your answer.

11. If the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^2}$ is approximated with the series $\sum_{n=1}^{7}(-1)^{n+1}\frac{1}{n^2}$, what is the error bound?