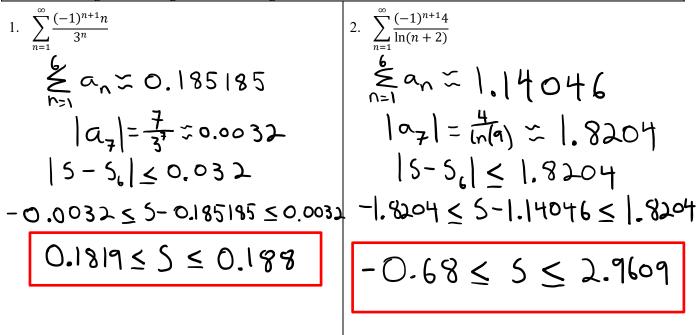
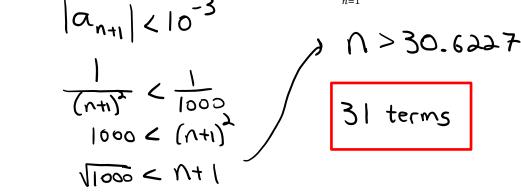
## **10.10 Alternating Series Error Bound**

## Calculus

A calculator may be used on all problems in this practice. For 1-2, approximate an interval of the sum of the alternating series using the Alternating Series Error Bound for the first 6 terms.

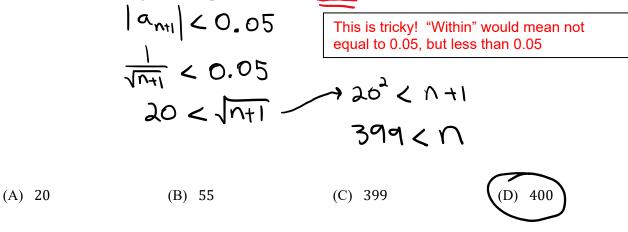


3. Determine the number of terms needed to approximate the sum of the series  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$  with an error less than  $10^{-3}$ .



4. The series  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$  converges to S. Using the alternating series bound, what is the least number of terms

that must be summed to guarantee a partial sum that is within 0.05 of S?



5. If the infinite series 
$$S = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{4}{n}$$
 is approximated by  $P_k = \sum_{n=1}^{k} (-1)^{n+1} \frac{4}{n}$ , what is the least value of k for which the alternating series error bound guarantees that  $|S - P_k| < \frac{7}{100}$ ?  
 $\left| 5 - \rho_k \right| \leq \left| \alpha_{k+1} \right| < \frac{7}{100}$ 
 $k > 56.1428$   
 $\frac{4}{k+1} < \frac{7}{100}$ 
 $k > 56.1428$   
(A) 55 (B) 56 (C) 57 (D) 60

6. If the series  $S = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^3}$  is approximated by the partial sum  $S_k = \sum_{n=1}^{k} (-1)^{n+1} \frac{1}{n^3}$ , what is the least value of k for which the alternating series error bound guarantees that  $|S - S_k| \le \frac{7}{10000}$ ?

$$\begin{aligned} S-S_{k} &| \leq |\alpha_{k+1}| \leq \frac{7}{10,000} \\ & \left(\frac{1}{(k+1)^{3}} \leq \frac{7}{10,000} \\ & \left(\frac{10,000}{7} \leq (k+1)^{3}\right) \\ & 3\sqrt{10,000} \leq (k+1) \\ & 3\sqrt{10,000} \leq (k+1) \end{aligned} \qquad (B) 11 \qquad (C) 12 \qquad (D) 13 \end{aligned}$$

7. The series  $\sum_{k=1}^{\infty} (-1)^{k+1} a_k$  converges by the alternating series test. If  $S_n = \sum_{k=1}^n (-1)^{k+1} a_k$  is the *n*th partial sum of the series, which of the following statements must be true?

(A) 
$$\lim_{n \to \infty} S_n = 0$$
 (B)  $\lim_{n \to \infty} a_n = S$  (C)  $|S - S_{20}| \le a_{26}$  (D)  $|S - S_{25}| \le a_{26}$ 

8. If the series 
$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{5n+1}$$
 is approximated by the partials sum with 15 terms, what is the alternating series error bound?  

$$\left| \boldsymbol{\alpha}_{n+1} \right| = \frac{1}{5(n+1)+1} = \frac{1}{5(10)+1} = \frac{1}{5(10)+1} = \frac{1}{80+1}$$
(A)  $\frac{1}{15}$  (B)  $\frac{1}{16}$  (C)  $\frac{1}{76}$  (D)  $\frac{1}{81}$ 

9. The function f is defined by the power series  $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x}{(2n+1)!}$  for all real numbers x. Show that  $1 - \frac{1}{3!} + \frac{1}{5!}$  approximates f(1) with an error less than  $\frac{1}{4000}$ .

**Test Prep** 

## 10.10 Alternating Series Error Bound

10. Calculator active! Let  $f(x) = \sum_{n=1}^{\infty} \frac{x^n n^n}{n!}$  for all x for which the series converges.

a. Use the first three terms of the series to approximate  $f\left(-\frac{1}{3}\right)$ .

$$f(-\frac{1}{3}) \approx \sum_{n=1}^{3} \frac{(-\frac{1}{3})^{n} \cdot n^{n}}{n!} = -0.2778$$

b. How far off is this estimate from the value of  $f\left(-\frac{1}{3}\right)$ ? Justify your answer.

Error bound is the next term.  

$$a_{4} = \frac{(-\lambda_{3}) \cdot \mu^{4}}{4!} \approx 0.13168$$
The estimate of -0.2778 is off by 0.13168  
or less. The error bound is the next term  $a_{4}$ .  
11. If the series  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^{2}}$  is approximated with the series  $\sum_{n=1}^{7} (-1)^{n+1} \frac{1}{n^{2}}$ , what is the error bound?  
Error bound = 8<sup>th</sup> term  $7^{th}$  term  
 $|a_{8}| = \frac{1}{8^{2}} = \frac{1}{64}$