10.12 Lagrange Error Bound

Calculus Name: ______________________________ **CA #1**

1. The fourth-degree Maclaurin polynomial for $\cos x$ is given by $1 - \frac{x^2}{2!} + \frac{x^4}{4!}$. Use the Lagrange error bound to estimate the error in using this polynomial to approximate $\cos\frac{\pi}{3}$.

2. The function f has derivatives of all orders for all real numbers and $f^{(4)}(x) = e^{\sin x}$. If the third-degree Taylor Polynomial for f about $x = 0$ is used to approximate f on $[0,1]$, what is the Lagrange error bound for the maximum error on $[0,1]$?

3. Assume a third-degree Taylor Polynomial about $x = 2$ is used for the approximation f and $\left|f^{(4)}(x)\right| \leq 12$ for all $x \geq 1$. Which of the following represents the Lagrange error bound in the approximation of $f(2.5)$?

   (A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{16}$ (D) $\frac{1}{32}$

4. Determine the degree of the Taylor Polynomial about $x = 0$ for $f(x) = e^x$ required for the error in the approximation of $f(0.8)$ to be less than 0.005.

5.

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$	$f^{(4)}(x)$
2	112	164	214	312	345

Let f be a function having derivatives of all orders for $x > 0$. Selected values for the first four derivatives of f are given for $x = 2$. Use the Lagrange error bound to show that the third-degree Taylor Polynomial for f about $x = 2$ approximates $f(1.9)$ with an error less than 0.002.

Answers to 10.12 CA #1

1. 0.0105	2. 0.0967	3. D	4. $n = 5$	5. $R_3 = 0.0014375 < 0.002$