10.12 Lagrange Error Bound

Calculus Name: ______________________________ **CA #2**

1. The fifth-degree Maclaurin polynomial for $\sin x$ is given by $x - \frac{x^3}{3!} + \frac{x^5}{5!}$. Use the Lagrange error bound to estimate the error in using this polynomial to approximate $\sin\frac{\pi}{3}$.

2. The function f has derivatives of all orders for all real numbers and $f^{(3)}(x) = e^{\cos x}$. If the second-degree Taylor Polynomial for f about $x = 0$ is used to approximate f on $[0,1]$, what is the Lagrange error bound for the maximum error on $[0,1]$?

3. Assume a fourth-degree Taylor Polynomial about $x = 2$ is used for the approximation f and $\left|f^{(5)}(x)\right| \leq 12$ for all $x \geq 1$. Which of the following represents the Lagrange error bound in the approximation of $f(2.5)$?

(A) $\frac{1}{120}$ (B) $\frac{1}{320}$ (C) 12 (D) $\frac{3}{8}$

4. Determine the degree of the Taylor Polynomial about $x = 0$ for $f(x) = \sin x$ required for the error in the approximation of $f(0.3)$ to be less than 10^{-5}.

5.

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$	$f^{(4)}(x)$
1	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{9}{10}$	$\frac{13}{12}$	$\frac{16}{21}$

Let f be a function having derivatives of all orders for $x > 0$. Selected values for the first four derivatives of f are given for $x = 1$. Use the Lagrange error bound to show that the third-degree Taylor Polynomial for f about $x = 1$ approximates $f(0.8)$ with an error less than 10^{-4}.

Answers to 10.12 CA #2

1. 0.00183	2. 0.4530	3. B	4. $n = 5$	5. $R_3 = 5.079 \times 10^{-5} \leq 10^{-4}$