Calculus

Write your questions and thoughts here!

10.13 Radius and Interval of Convergence of Power Series

Notes

Power Series

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + \cdots + a_n x^n$$

$$\sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1 (x-c)^1 + a_2 (x-c)^2 + a_3 (x-c)^3 + \cdots + a_n (x-c)^n$$

The domain of a power series is the set of all x-values for which the power series converges.

Note! The center is always part of the domain.

Three ways a power series may converge:

1.
   a.
2.
3.

The **Interval of Convergence** is the set of values for convergence. We use the Ratio Test to find the interval of convergence.

Ratio Test for Interval of Convergence

If you have a power series $\sum_{n=1}^{\infty} a_n$, find $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$.

- $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$, then the series converges
- $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 0$, then the series converges
- $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \infty$, then the series converges

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Find the radius and interval of convergence.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{3^n}(x-5)^n$

2. $\displaystyle\sum_{n=0}^{\infty} 3(x-2)^n$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^{n+1}}{n^3}$

4. $\displaystyle\sum_{n=0}^{\infty} \frac{(2n)!\,x^{2n}}{n!}$

5. $\displaystyle\sum_{n=0}^{\infty} \frac{x^{3n}}{n!}$

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Practice

Find the interval of convergence for each power series.

1. $\displaystyle\sum_{n=0}^{\infty} \frac{(x-1)^n}{4^n}$

2. $\displaystyle\sum_{n=0}^{\infty} \frac{(x+2)^n}{3^n}$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-2)^n}{n2^n}$

4. $\displaystyle\sum_{n=0}^{\infty} (2n)!\left(\frac{x}{3}\right)^n$

Find the radius of convergence for each series.

5. $\displaystyle\sum_{n=1}^{\infty}\frac{(4x)^n}{n^2}$

6. $\displaystyle\sum_{n=0}^{\infty}\frac{(x-4)^{n+1}}{2\cdot 3^{n+1}}$

7. $\displaystyle\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!}$

8. $\displaystyle\sum_{n=0}^{\infty}\frac{(2n)!\ x^{2n}}{n!}$

What are all values of x for which each series converges?

9. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{4}{x^2+1}\right)^n$

10. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\left(x+\frac{3}{2}\right)^n$

11. $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n \cdot 3^n}$

12. $\sum_{n=0}^{\infty} \frac{x^{5n}}{n!}$

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Test Prep

13. The radius of convergence for the power series $\sum_{n=1}^{\infty} \frac{(x-4)^{2n}}{n}$ is equal to 1. What is the interval of convergence?

14. If the power series $\sum_{n=0}^{\infty} a_n (x-5)^n$ converges at $x = 8$ and diverges at $x = 10$, which of the following must be true?

I. The series converges at $x = 2$.

II. The series converges at $x = 3$.

III. The series diverges at $x = 0$.

(A) I only (B) II only (C) I and II only (D) II and III only

15. The coefficients of the power series $\sum_{n=0}^{\infty} a_n(x-3)^n$ satisfy $a_0 = 6$ and $a_n = \left(\frac{2n+1}{3n+1}\right) a_{n-1}$ for all $n \geq 1$. What is the radius of convergence?

16. The radius of convergence for the power series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-5)^n}{n5^n}$ is 5, what is the interval of convergence?

(A) $-5 < x < 5$ (B) $-5 < x \leq 5$ (C) $0 < x < 10$ (D) $0 < x \leq 10$

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17. Let $a_n = \frac{1}{n \ln n}$ for $n \geq 3$ and let f be the function given by $f(x) = \frac{1}{x \ln x}$.

a. The function f is continuous, decreasing, and positive. Use the Integral Test to determine the convergence or divergence of the series $\sum_{n=3}^{\infty} a_n$.

b. Find the interval of convergence of the power series $\sum_{n=3}^{\infty} \frac{(x-2)^{n+1}}{n \ln n}$.