## Calculus

Write your questions and thoughts here!

Power Series

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + \dots + a_n x^n$$
$$\sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1 (x-c)^1 + a_2 (x-c)^2 + a_3 (x-c)^3 + \dots + a_n (x-c)^n$$

The domain of a power series is the set of all *x*-values for which the power series converges.

Note! The center is always part of the domain.

Three ways a power series may converge:

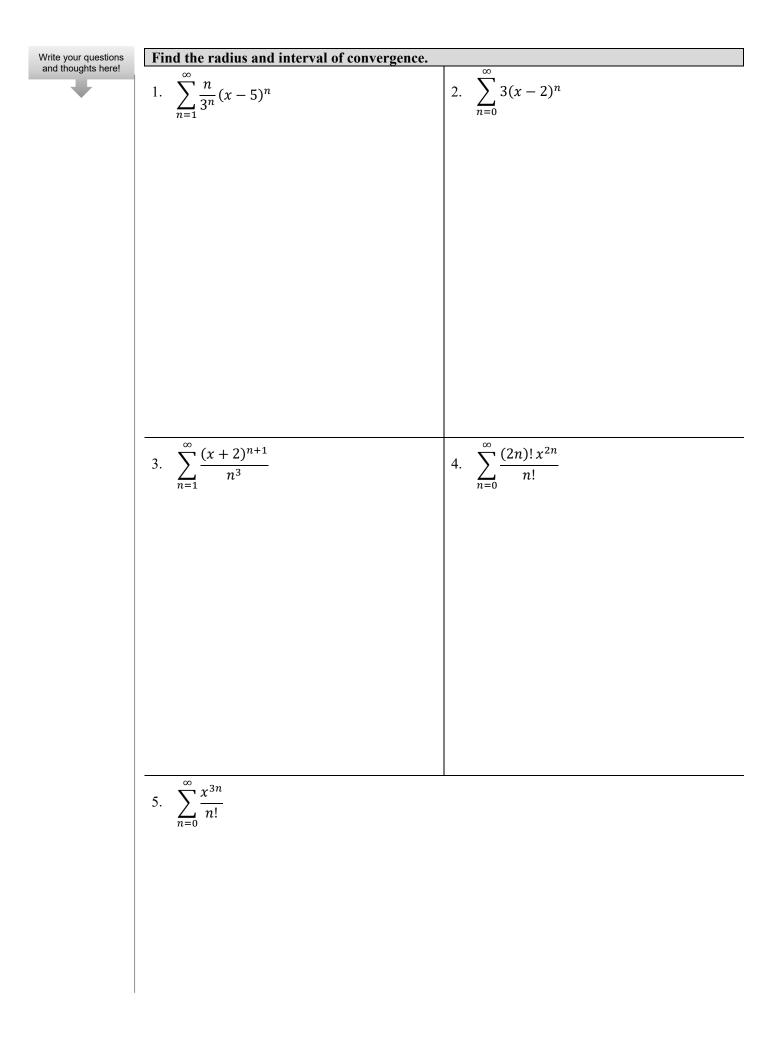
- 1. a. 2.
- ----
- 3.

The **Interval of Convergence** is the set of values for convergence. We use the Ratio Test to find the interval of convergence.

## Ratio Test for Interval of Convergence

If you have a power series  $\sum_{n=1}^{\infty} a_n$ , find  $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$ .

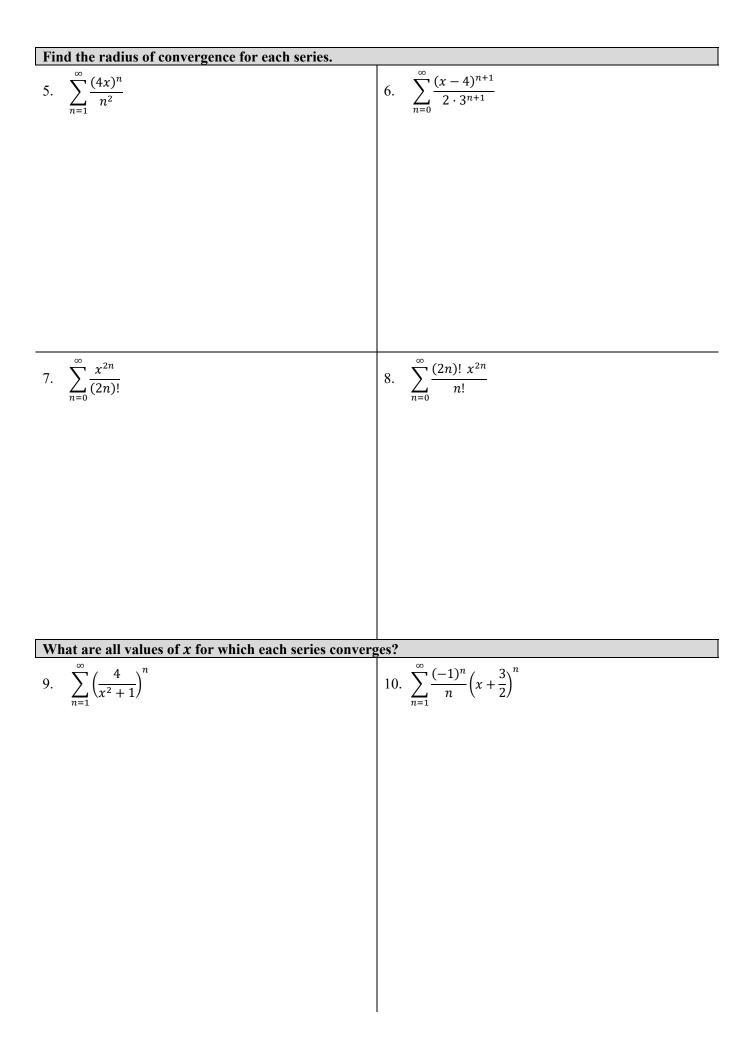
- $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$ , then the series converges
- $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = 0$ , then the series converges
- $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \infty$ , then the series converges



## **10.13 Radius and Interval of Convergence of Power Series** Calculus

Practice

Calculus	
Find the interval of convergence for each power series.	
$1.  \sum_{n=0}^{\infty} \frac{(x-1)^n}{4^n}$	$2. \qquad \sum_{n=0}^{\infty} \frac{(x+2)^n}{3^n}$
3. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-2)^n}{n2^n}$	$4.  \sum_{n=0}^{\infty} (2n)! \left(\frac{x}{3}\right)^n$



$$11. \quad \sum_{n=1}^{\infty} \frac{(x-2)^n}{n \cdot 3^n}$$

12. 
$$\sum_{n=0}^{\infty} \frac{x^{5n}}{n!}$$

## 10.13 Radius and Interval of Convergence of Power Series

13. The radius of convergence for the power series  $\sum_{n=1}^{\infty} \frac{(x-4)^{2n}}{n}$  is equal to 1. What is the interval of convergence?

14. If the power series  $\sum_{n=0}^{\infty} a_n (x-5)^n$  converges at x = 8 and diverges at x = 10, which of the following must be true?

- I. The series converges at x = 2.
- II. The series converges at x = 3.
- III. The series diverges at x = 0.

(A) I only

(B) II only

(C) I and II only

15. The coefficients of the power series  $\sum_{n=0}^{\infty} a_n (x-3)^n$  satisfy  $a_0 = 6$  and  $a_n = \left(\frac{2n+1}{3n+1}\right) a_{n-1}$  for all  $n \ge 1$ . What is the radius of convergence?

16. The radius of convergence for the power series  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-5)^n}{n5^n}$  is 5, what is the interval of convergence?

(A) 
$$-5 < x < 5$$
 (B)  $-5 < x \le 5$  (C)  $0 < x < 10$  (D)  $0 < x \le 10$ 

17. Let  $a_n = \frac{1}{n \ln n}$  for  $n \ge 3$  and let *f* be the function given by  $f(x) = \frac{1}{x \ln x}$ .

a. The function f is continuous, decreasing, and positive. Use the Integral Test to determine the convergence or divergence of the series  $\sum_{n=3}^{\infty} a_n$ .

b. Find the interval of convergence of the power series  $\sum_{n=3}^{\infty} \frac{(x-2)^{n+1}}{n \ln n}$