Calculus

10.14 Finding Taylor or Maclaurin Series

Notes

Write your questions and thoughts here!

Taylor Series

If $f(x)$ has derivatives of all orders at $x = c$, then a Taylor Series may be formed that is equal to the function for many common functions.

If $c = 0$ it is a

You need to know the following series:

The Taylor series of these functions are exact when we go to ∞. They must be memorized!

Maclaurin Series for e^x.	Maclaurin Series for $\sin x$.

Memorize the following!

Function	Series (expanded)	Series Notation	Int. of Conv.
$e^x =$			
$\sin x =$			
$\cos x =$			
$\frac{1}{1+x} =$			

Write your questions and thoughts here!

The function $f(x) = \frac{1}{1-x}$ is actually a geometric series.

Recall: $\sum_{n=k}^{\infty} ar^n =$

Find the Maclaurin Series for each of the following functions.

3. $\sin x^2$

4. $x^2 e^x$

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Practice

1. What is the coefficient of x^6 in the Taylor Series about $x = 0$ for the function $f(x) = \frac{e^{2x^2}}{2}$?

2. If $f(x) = x \sin 3x$, what is the Taylor Series for f about $x = 0$? Write the first four non-zero terms.

3. What is the Maclaurin Series for $\frac{1}{(1-x)^2}$? Write the first four non-zero terms.

4. What is the Maclaurin Series for the function $f(x) = \frac{1}{2}(e^x + e^{-x})$? Write the first four non-zero terms.

5. Find the Maclaurin Series for the function $f(x) = \cos\sqrt{x}$. Write the first four non-zero terms.

6. Find the Maclaurin Series for the function $f(x) = \sin 5x$. Write the first four non-zero terms.

7. What is the Taylor series expansion about $x = 0$ for the function $f(x) = \frac{\sin x}{x}$? Write the first four non-zero terms.

8. The sum of the series $1 + \frac{3}{1!} + \frac{3^2}{2!} + \frac{3^3}{3!} + \cdots + \frac{3^n}{n!}$ is

(A) $\ln 3$ (B) e^3 (C) $\cos 3$ (D) $\sin 3$

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9. What is the sum of the series $1 + \ln 3 + \frac{(\ln 3)^2}{2!} + \cdots + \frac{(\ln 3)^n}{n!}$?

10. What is the Taylor Series about $x = 0$ for the function $f(x) = 1 + x^2 + \cos x$? Write the first four non-zero terms.

11. What is the sum of the infinite series $1 - \left(\frac{\pi}{2}\right)^2\left(\frac{1}{3!}\right) + \left(\frac{\pi}{2}\right)^4\left(\frac{1}{5!}\right) - \left(\frac{\pi}{2}\right)^6\left(\frac{1}{7!}\right) + \cdots + \frac{\left(\frac{\pi}{2}\right)^{2n}(-1)^n}{(2n+1)!}$?

12. Find the Maclaurin Series for the function $f(x) = e^{-3x}$. Write the first four non-zero terms.

13. Find the Maclaurin Series for the function $f(x) = \frac{\sin x^2}{x} + \cos x$. Write the first four non-zero terms.

14. Which of the following is the Maclaurin Series for the function $f(x) = x\cos 2x$?

(A) $\sum_{n=0}^{\infty} \frac{(-1)^n 2x^{2n}}{(2n)!}$

(B) $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{2n}}{(2n)!}$

(C) $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{2n+1}}{(2n)!}$

(D) $\sum_{n=0}^{\infty} \frac{(-1)^n 2x^{2n+1}}{(2n)!}$

15. The Maclaurin series $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots + \frac{x^{2n+1}}{(2n+1)!}$ represents which function $f(x)$

(A) $\sin x$ (B) $-\sin x$ (C) $\frac{1}{2}(e^x - e^{-x})$ (D) $e^x - e^{-x}$

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16. The function f satisfies the equation $f'(x) = f(x) + x + 1$ and $f(0) = 2$. The Taylor Series for f about $x = 0$ converges to $f(x)$ for all x.

a. Write an equation for the line tangent to the curve of $y = f(x)$ at $x = 0$.

b. Find $f''(0)$ and find the second-degree Taylor Polynomial for f about $x = 0$.

c. Find the fourth-degree Taylor Polynomial for f about $x = 0$.

d. Find $f^{(n)}(0)$, the n^{th} derivative of f about $x = 0$, for $n \geq 2$. Use the Taylor Series for f about $x = 0$ and the Taylor Series for e^x about $x = 0$ to find $f(x) - 4e^x$.