10.15 Representing Functions as Power Series

Notes

Recall:

Function	Series (expanded)	Series Notation	Int. of Conv.
$e^x =$	$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$	$\sum_{n=0}^{\infty} \frac{x^n}{n!}$	$-\infty < x < \infty$
$\sin x =$	$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$	$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$	$-\infty < x < \infty$
$\cos x =$	$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$	$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$	$-\infty < x < \infty$
$\frac{1}{1+x} =$	$1 - x + x^2 - x^3 + \cdots$	$\sum_{n=0}^{\infty} (-1)^n x^n$	$-1 < x < 1$

1. If $f(x) = \sum_{n=0}^{\infty} \frac{x^{5n}}{n!}$ then $f'(x) =$

2. Write the first 4 nonzero terms for the Maclaurin series that represents $\int_0^x \sin(t^7)\, dt$.

Practice

1. What is the coefficient of x^2 in the Taylor Series for the function $f(x) = \sin^2 x$ about $x = 0$?

2. If the function f is defined as $f(x) = \sum_{n=1}^{\infty} \frac{x^{2n}}{n!}$, then what is $f'(x)$? Write the first four nonzero terms and the general term.

3. Use the power series expansion for $\cos x^6$ to evaluate the integral $\int_0^x \cos t^6 \, dt$. Write the first four nonzero terms and the general term.

4. For $x > 0$, the power series defined by $1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots + \frac{(-1)^n x^{2n}}{(2n+1)!}$ converges to which of the following?

(A) $\cos x$ (B) $\sin x$ (C) $\frac{\sin x}{x}$ (D) $e^x - e^{x^2}$

5. It is known that the Maclaurin series for $\frac{1}{1-x}$ is $\sum_{n=0}^{\infty} x^n$. Use this fact to assist in finding the first four nonzero terms and the general term for the power series expansion for the function $\frac{x^2}{1-x^2}$.

6. Let f be the function with initial condition $f(0) = 0$ and derivative $f'(x) = \frac{1}{1+x^7}$. Write the first four nonzero terms of the Maclaurin series for the function f.

7. Find the Maclaurin series for the function $f(x) = e^{3x}$. Write the first four nonzero terms and the general term.

8. If a function has the derivative $f'(x) = \sin(x^2)$ and initial conditions $f(0) = 0$, write the first four nonzero terms of the Maclaurin series for f.

9. The function f has derivatives of all orders and the Maclaurin series for the function f is given by $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+3}$. Find the Maclaurin series for the derivative $f'(x)$. Write the first four nonzero terms and the general term.

10. Let the function f be defined by $f(x) = \frac{1}{1-x}$. Find the Maclaurin series for the derivative f'. Write the first four nonzero terms and the general term.

11. Find the second-degree Taylor Polynomial for the function $f(x) = \frac{\cos x}{1-x}$ about $x = 0$.

12. What is the coefficient of x^2 in the Maclaurin series for the function $f(x) = \left(\frac{1}{1+x}\right)^2$?

13. Find the Maclaurin series for the function $f(x) = x \cos x^2$. Write the first four nonzero terms and the general term.

14. Given that f is a function that has derivatives of all orders and $f(1) = 3, f'(1) = -2, f''(1) = 2$, and $f'''(1) = 4$. Write the second-degree Taylor Polynomial for the derivative f' about $x = 1$ and use it to find the approximate value of $f'(1.2)$.

15. Let the fourth-degree Taylor Polynomial be defined by $T = 7 - 3(x-4) + 5(x-4)^2 - 2(x-4)^3 + 6(x-4)^4$ for the function f about $x = 4$. Find the third-degree Taylor Polynomial for f' about $x = 4$ and then use it to approximate $f'(4.2)$.

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Test Prep

16. Given a function defined by $f(x) = \frac{\cos(2x)-1}{x^2}$ for $x \neq 0$ and is continuous for all real numbers x.
    a. What is the limit of the function $f(x)$ as x approaches 0?

b. Write the first four nonzero terms and the general term of the power series that represents the function $h(x) = \cos 2x$

c. Use the results from part (b) to write the first three nonzero terms for $f(x) = \frac{\cos(2x)-1}{x^2}$.

d. Use the results from part (c) to determine if the function $f(x) = \frac{\cos(2x)-1}{x^2}$ has a relative maximum, a relative minimum or neither at $x = 0$. Justify your answer.