10.1 Convergent and Divergent Infinite Series



Practice

1. Given the infinite series
$$\sum_{n=1}^{\infty} (-1)^n$$
, find the sequence of partial sums S_1, S_2, S_3, S_4 , and S_5 .

$$S_1 = \frac{1}{1} \qquad S_3 = \frac{1}{1} = \frac{1}{1} \qquad S_5 = \frac{1}{1} = \frac{1}{1}$$

2. Find the sequence of partial sums S_1, S_2, S_3, S_4 , and S_5 for the infinite series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10} + \cdots$. $S_1 = 1$ $S_3 = \frac{3}{4} + \frac{1}{4} = \frac{3}{4} + \frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10} + \cdots$

$$5_1 = 1$$

 $5_2 = \frac{3}{2}$

$$S_1 = 1$$
 $S_3 = \frac{3}{4} + \frac{1}{4} = \frac{7}{4}$
 $S_2 = \frac{3}{4} = \frac{7}{4} + \frac{1}{4} = \frac{7}{4}$

3. If the infinite series $\sum_{n=1}^{\infty} a^n$ has nth partial sum $S_n = (-1)^{n+1}$ for $n \ge 1$, what is the sum of the series? $\sum_{n=1}^{\infty} a^n$ diverges

$$\lim_{n\to\infty} \int_{n}^{n=1}$$

4. The infinite series $\sum_{n=0}^{\infty} a^n$ has *n*th partial sum $S_n = \frac{n}{4n+1}$ for $n \ge 1$. What is the sum of the series?

5. Use a calculator to find the partial sum S_n of the series $\sum_{n=0}^{\infty} \frac{6}{n(n+3)}$ for n=100,500,1000.

6. Show that the sequence with the given *n*th term $a_n = 1 + 2n$ is monotonic.

$$\alpha_{1}=3$$
 $\alpha_{2}=5$ $\alpha_{3}=7$ $\alpha_{4}=9$...

an is monotonic because a £ a £ a £ a ... Lan

7. What is the *n*th partial sum of the infinite series $\sum \frac{1}{2^{n+1}}$?

$$S_{n} = \frac{1}{2^{2}} + \frac{1}{2^{3}} + \frac{1}{2^{4}} + \cdots + \frac{1}{2^{m+1}}$$

$$\frac{1}{2^{2}} = \frac{1}{2^{2}} + \frac{1}{2^{4}} + \frac{1}{2^{5}} + \cdots + \frac{1}{2^{m+2}}$$

$$S_{n} - \frac{1}{2^{2}} = \frac{1}{2^{2}} + \frac{1}{2^{4}} + \frac{1}{2^{5}} + \cdots + \frac{1}{2^{m+2}}$$

$$5_{n} = \frac{1}{2^{2}} - \frac{1}{2^{n+2}}$$

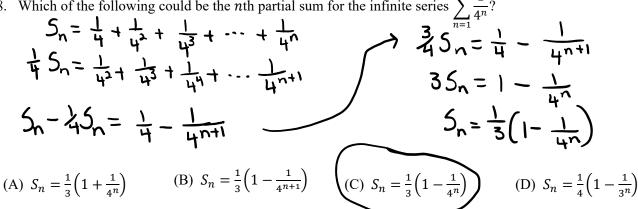
$$5_{n} = 2\left[\frac{1}{2^{2}} - \frac{1}{2^{n+2}}\right]$$

$$5_{n} = \frac{1}{2} - \frac{1}{2^{n+1}}$$

10.1 Convergent and Divergent Infinite Series



8. Which of the following could be the *n*th partial sum for the infinite series $\sum_{n=0}^{\infty} \frac{1}{4^n}$?



9. If the infinite series $\sum_{n=0}^{\infty} a_n$ is convergent and has a sum of $\frac{7}{8}$, which of the following could be the *n*th partial

(A)
$$S_n = \frac{7n+1}{8n^2+1}$$
 \longrightarrow \bigcirc

(B)
$$S_n = \frac{7n^2+1}{8n+1} \longrightarrow \infty$$

(C)
$$S_n = 2\left(\frac{7}{8} - \frac{1}{n+2} - \frac{1}{n+3}\right) \longrightarrow \frac{14}{8}$$

(D)
$$S_n = \left(\frac{7}{8} - \frac{1}{n+2} - \frac{1}{n+3}\right) \rightarrow \frac{7}{8}$$

10. Which of the following sequences with the given nth term is bounded and monotonic?

