Calculus

10.2 Working with Geometric Series

Notes

Write your questions and thoughts here!

Recall: What is a geometric sequence?
A **geometric sequence** is one in which the same number is   to each term to get the next term in the **sequence**. The number you multiply by is called the   usually denoted by

n^{th} Term of a Geometric Sequence

The nth term of a geometric sequence with first term a_1 and common ratio r is given by:

$$a_n = \qquad \text{or} \quad a_n = \qquad \text{or} \quad a_n =$$

1. $3, 6, 12, 24, 48, \ldots$

$a_n = a_0 r^n$	$a_n = a_1 r^{n-1}$	$a_n = a_2 r^{n-2}$
$a_n =$	$a_n =$	$a_n =$

2. $25, 5, 1, \frac{1}{5}, \frac{1}{25}, \ldots$

$a_n = a_0 r^n$	$a_n = a_1 r^{n-1}$	$a_n = a_2 r^{n-2}$
$a_n =$	$a_n =$	$a_n =$

$$\sum_{n=0}^{\infty} ar^n =$$

$a \neq 0$

Geometric Infinite Series Convergence

A geometric series with ratio r diverges when   If   then the series converges to

$$\sum_{n=k}^{\infty} ar^n =$$

Where   is the first term of the series.

1. $\sum_{n=0}^{\infty} \frac{3}{4^n}$

2. $\sum_{n=2}^{\infty} \frac{3^{n+1}}{4^n}$

Write your questions and thoughts here!

3. For what value of r does the infinite series $\sum_{n=0}^{\infty} 17r^n$ equal 23?

4. **Calculator active.** If $f(x) = \sum_{n=3}^{\infty} \left(\sin^2\left(\frac{x}{3}\right)\right)^n$, then $f(7) =$

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Practice

Find the value of each infinite series.

1. $\sum_{n=1}^{\infty} -\frac{7}{(-3)^n}$

2. $\sum_{n=0}^{\infty} \frac{1}{3^n}$

3. $\sum_{n=0}^{\infty} e^{nx}$ Let x be a real number, with $x < 0$.

4. $\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^n$

5. $\sum_{n=1}^{\infty} \frac{3^{n+1}}{5^n}$

6. $\sum_{n=1}^{\infty} \frac{2^n}{e^{n+1}}$

7. $\sum_{n=0}^{\infty} (-1)^n \frac{\pi}{e^{n+1}}$

8. $\sum_{n=0}^{\infty} \left(-\frac{3}{4}\right)^n$

9. What is the sum of the infinite series $25 + \ -5 + 1 + \ -\frac{1}{5} + \frac{1}{25} + \cdots$

10. **Calculator active.** If $f(x) = \sum_{n=1}^{\infty} (\sin^2 2x)^n$, then $f(3) =$

11. For what value of a does the infinite series

$$\sum_{n=0}^{\infty} a\left(\frac{2}{3}\right)^n = 14$$

12. Consider the geometric series $\sum_{n=1}^{\infty} a_n$ where $a_n > 0$.

The first term of the series $a_1 = 24$, and the third term $a_3 = 6$. What are possible values for a_2?

13. Consider the series $\sum_{n=1}^{\infty} a_n$. If $a_1 = 32$ and $\frac{a_{n+1}}{a_n} = \frac{1}{4}$ for all integers $n \geq 1$, then $\sum_{n=1}^{\infty} a_n =$

14. Use a geometric series to write $0.\overline{2}$ as the ratio of two integers.

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Test Prep

15. If x and y are positive real numbers, which of the following conditions guarantees the infinite series $\sum_{n=0}^{\infty} \frac{x^{n+1}}{y^{2n+1}}$ is geometric and converges?

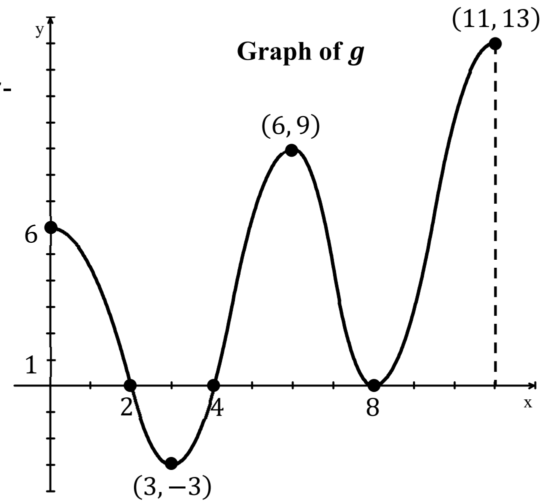
(A) $x < y$

(B) $x < y^2$

(C) $x > y^2$

(D) $x > y$

16. The figure to the right shows a portion of the graph of the differentiable function g. Let h be the function defined by $h(x) = \int_4^x g(t)\,dt$. The areas of the regions bounded by the x-axis and the graph of g on the intervals, [0,2], [2,4], [4,8] and [8,11] are $6, 4, 24$, and 19, respectively.



a. Must there exist a value of c, for $2 < c < 4$, such that $h(c) = 3.5$? Justify your answer.

b. Find the average value of g over the interval, $0 \le x \le 11$. Show the computations that lead to your answer.

c. Evaluate $\lim\limits_{x \to 8} \frac{h(x) - 3x}{x^2 - 64}$.

d. Is there a value r such that the series $30 + 30r + 30r^2 + \cdots + 30r^n$ equals the value of $g(6)$?