10.2 Working with Geometric Series

Calculus

Solutions

Practice

Find the value of each infinite series.

1. $\sum_{n=1}^{\infty} -\frac{7}{(-3)^n}$

$ar^k = \frac{7}{3}$ $\quad r = \left(-\frac{1}{3}\right)^n$

1st term

$|r| < 1$ so converges

$$\frac{7/3}{1 - -1/3} = \frac{7/3}{4/3} = \frac{7}{3} \cdot \frac{3}{4}$$

$$\boxed{7/4}$$

2. $\sum_{n=0}^{\infty} \frac{1}{3^n}$

$ar^k = 1$ $\quad r = \left(\frac{1}{3}\right)^n$

$|r| < 1$ so it converges

$$\frac{1}{1 - 1/3} = \frac{1}{2/3} = \boxed{3/2}$$

$(e^n)^z$

3. $\sum_{n=0}^{\infty} e^{nx}$ Let x be a real number, with $x < 0$.

$ar^k = 1$ $\quad r = e^x$

$|r| < 1$, converges

$$\boxed{\frac{1}{1 - e^x}}$$

4. $\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^n$

$ar^k = \frac{e}{\pi}$ $\quad r = \frac{e}{\pi}$

$|r| < 1$, converges

$$\frac{e/\pi}{1 - e/\pi} = \boxed{\frac{e}{\pi - e}}$$

5. $\sum_{n=1}^{\infty} \frac{3^{n+1}}{5^n}$

$$\frac{3^n \cdot 3^1}{5^n} = 3\left(\frac{3}{5}\right)^n$$

$ar^k = 9/5 \qquad r = 3/5$

$|r|<1$, so it converges

$$\frac{9/5}{1-3/5} = \frac{9/5}{2/5} = \frac{9}{5}\cdot\frac{5}{2} = \boxed{9/2}$$

6. $\sum_{n=1}^{\infty} \frac{2^n}{e^{n+1}}$

$$\frac{2^n}{e^n \cdot e^1} = \frac{1}{e}\left(\frac{2}{e}\right)^n$$

$ar^k = \frac{2}{e^2} \qquad r = 2/e$

$|r|<1$, so it converges

$$\frac{\frac{2}{e^2}}{1-2/e} \cdot \frac{e^2}{e^2} = \boxed{\frac{2}{e^2-2e}}$$

7. $\sum_{n=0}^{\infty} (-1)^n \frac{\pi}{e^{n+1}}$

$$\frac{\pi(-1)^n}{e^n \cdot e^1} = \frac{\pi}{e}\left(-\frac{1}{e}\right)^n$$

$|r|<1$

$ar^k = \frac{\pi}{e} \qquad r = -1/e$

$$\frac{\frac{\pi}{e}}{1+1/e} \cdot \frac{e}{e} = \boxed{\frac{\pi}{e+1}}$$

8. $\sum_{n=0}^{\infty} \left(-\frac{3}{4}\right)^n$

$ar^k = 1 \qquad r = -3/4$

$|r|<1$

$$\frac{1}{1+3/4} = \frac{1}{7/4}$$

$$\boxed{4/7}$$

9. What is the sum of the infinite series

$25 + -5 + 1 + -\frac{1}{5} + \frac{1}{25} + \cdots$

First term $= 25 \qquad r = -1/5$

$$\frac{25}{1+1/5} = \frac{25}{6/5} = 25 \cdot 5/6$$

$$\boxed{125/6}$$

10. **Calculator active.** If $f(x) = \sum_{n=1}^{\infty} (\sin^2 2x)^n$, then $f(3) =$

$ar^k = (\sin^2(6))^1 \approx 0.078073$

$r = [\sin^2(6)] \approx$ store b

$$\frac{b}{1-b} \approx \boxed{0.0846}$$

11. For what value of a does the infinite series

$$\sum_{n=0}^{\infty} a\left(\frac{2}{3}\right)^n = 14$$

$ar^k = a$ $r = \frac{2}{3}$

$\frac{a}{1-\frac{2}{3}} = 14$

$\frac{a}{\frac{1}{3}} = 14$

$\boxed{a = \frac{14}{3}}$

12. Consider the geometric series $\sum_{n=1}^{\infty} a_n$ where $a_n > 0$.

The first term of the series $a_1 = 24$, and the third term $a_3 = 6$. What are possible values for a_2?

$24 \cdot r^2 = 6$

$r^2 = \frac{1}{4}$

$r = \pm\frac{1}{2}$

$\boxed{a_2 = 24 \cdot (\tfrac{1}{2}) = 12 \text{ or } a_2 = 24(-\tfrac{1}{2}) = -12}$

13. Consider the series $\sum_{n=1}^{\infty} a_n$. If $a_1 = 32$ and $\frac{a_{n+1}}{a_n} = \frac{1}{4}$ for all integers $n \geq 1$, then $\sum_{n=1}^{\infty} a_n =$

This is r.

$\frac{32}{1-\frac{1}{4}} = \frac{32}{\frac{3}{4}} = 32 \cdot \frac{4}{3} = \boxed{\frac{128}{3}}$

14. Use a geometric series to write $0.\overline{2}$ as the ratio of two integers.

$0.\overline{2} = 0.2 + 0.02 + 0.002 \dots$

$= \frac{2}{10} + \frac{2}{100} + \frac{2}{1000} + \dots$

$= \frac{2}{10^1} + \frac{2}{10^2} + \frac{2}{10^3} + \dots$

First term $= \frac{2}{10}$ $r = \frac{1}{10}$

$\frac{\frac{2}{10}}{1-\frac{1}{10}} = \frac{\frac{1}{5}}{\frac{9}{10}} = \frac{1}{5} \cdot \frac{10}{9} = \boxed{\frac{2}{9}}$

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Test Prep

15. If x and y are positive real numbers, which of the following conditions guarantees the infinite series $\sum_{n=0}^{\infty} \frac{x^{n+1}}{y^{2n+1}}$ is geometric and converges?

$\frac{x^n \cdot x^1}{y^{2n} \cdot y^1} = \frac{x}{y}\left(\frac{x}{y^2}\right)^n$

r

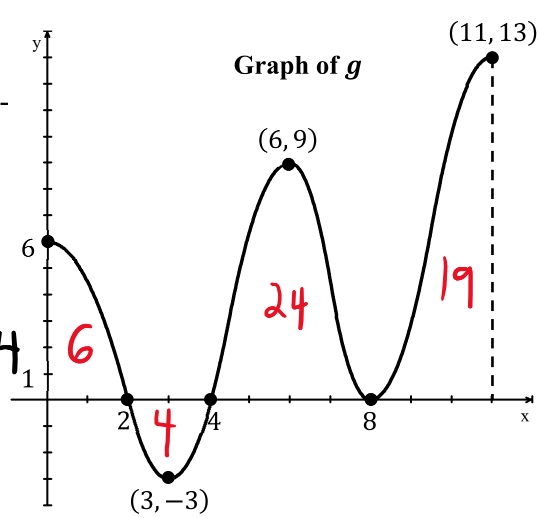
r

$\frac{x}{y^2} < 1$

$x < y^2$

(A) $x < y$ (B) $x < y^2$ (C) $x > y^2$ (D) $x > y$

16. The figure to the right shows a portion of the graph of the differentiable function g. Let h be the function defined by $h(x) = \int_4^x g(t)\,dt$. The areas of the regions bounded by the x-axis and the graph of g on the intervals, [0,2], [2,4], [4,8] and [8,11] are 6, 4, 24, and 19, respectively.



a. Must there exist a value of c, for $2 < c < 4$, such that $h(c) = 3.5$? Justify your answer.

$h(2) = \int_4^2 g(t)\,dt = -\int_2^4 g(t)\,dt = -(-4) = 4$

$h(4) = \int_4^4 g(t)\,dt = 0$

$h(4) < 3.5 < h(2)$

$\boxed{\text{Yes, by the IVT}}$

b. Find the average value of g over the interval, $0 \le x \le 11$. Show the computations that lead to your answer.

$\frac{1}{11}\int_0^{11} g(x)\,dx = \frac{1}{11}[6 + -4 + 24 + 19]$

$\frac{1}{11}[45]$

$\boxed{45/11}$

c. Evaluate $\lim_{x\to 8} \frac{h(x)-3x}{x^2-64}$.

$h(8) = \int_4^8 g = 24$

$\lim_{x\to 8} \frac{24-3x}{x^2-64} = \frac{0}{0}$

L'Hopital's

$\lim_{x\to 8} \frac{-3}{2x} = -3/16$

d. Is there a value r such that the series $\underbrace{30 + 30r + 30r^2 + \cdots + 30r^n}$ equals the value of $\underline{g}(6)$?

$g(6) = 9$

geometric only converges if $|r| < 1$.

g not h!

First term $= 30$

$9 = \frac{30}{1-r}$

$9 - 9r = 30$

$-9r = 21$

$r = -7/3$

No, because this value of r would cause the series to diverge.