10.3 The nth Term Test for Divergence

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots + a_n$$

If $\displaystyle\sum_{n=1}^{\infty} a_n$ converges, then $\displaystyle\lim_{n\to\infty} a_n =$

If $\displaystyle\lim_{n\to\infty} a_n = 0$, then $\displaystyle\sum_{n=1}^{\infty} a_n$

Nth Term Test for Divergence

If $\displaystyle\lim_{n\to\infty} a_n \neq 0$, then

Use the Nth term test to make a conclusion about divergence for each series.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{3n^3 + 1}{5n^3 - 2n^2 + 1}$

2. $\displaystyle\sum_{n=0}^{\infty} 3\left(\frac{1}{2}\right)^n$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

4. $\displaystyle\sum_{n=1}^{\infty} \frac{2^{n+2}}{2^{n+3} + 1}$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{e^{4n}}{3n}$

For each of the following series, determine the convergence or divergence of the given series. State the reasoning behind your answer.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{3-2n}{5n+1}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n+1}}{5^n}$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{\sqrt{n^2+1}}$

4. $\displaystyle\sum_{n=1}^{\infty} \frac{e^{n+1}}{\pi^n}$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{7^n+1}{7^{n+1}}$

6. $\displaystyle\sum_{n=0}^{\infty} 5\left(\frac{5}{2}\right)^n$

7. The nth-Term Test can be used to determine divergence for which of the following series?

   I. $\displaystyle\sum_{n=1}^{\infty} \sin 2n$ II. $\displaystyle\sum_{n=1}^{\infty}\left(2 + \frac{3}{n}\right)$ III. $\displaystyle\sum_{n=1}^{\infty} \frac{n^3 + 1}{n^2}$

   (A) II only   (B) III only   (C) I and II only   (D) I, II, and III

8. The nth-Term Test can be used to determine divergence for which of the following series?

   I. $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n-1}{n}\right)$ II. $\displaystyle\sum_{n=1}^{\infty} \frac{3n - 2n^2}{5n^2}$ III. $\displaystyle\sum_{n=1}^{\infty} 3\left(\frac{5}{4}\right)^n$

   (A) II only   (B) II and III only   (C) I and II only   (D) I, II, and III

9. If $a_n = \cos\left(\frac{\pi}{2n}\right)$ for $n = 1,2,3, \ldots$, which of the following about $\displaystyle\sum_{n=1}^{\infty} a_n$ must be true?

   (A) The series converges and $\displaystyle\lim_{n\to\infty} a_n = 0$.

   (B) The series diverges and $\displaystyle\lim_{n\to\infty} a_n = 0$

   (C) The series diverges and $\displaystyle\lim_{n\to\infty} a_n \neq 0$

   (D) The series converges and $\displaystyle\lim_{n\to\infty} a_n \neq 0$