10.3 The nth Term Test for Divergence

Calculus

Solutions

Practice

For each of the following series, determine the convergence or divergence of the given series. State the reasoning behind your answer.

1. $\sum_{n=1}^{\infty} \frac{3-2n}{5n+1}$

$\lim_{n\to\infty} \frac{3-2n}{5n+1} = \boxed{-\frac{2}{5}}$

Diverges by the n^{th} Term Test.

2. $\sum_{n=1}^{\infty} \frac{3^{n+1}}{5^n}$

$\frac{3^n \cdot 3}{5^n}$

$\lim_{n\to\infty} 3\left(\frac{3}{5}\right)^n = 0$

Geometric! $|r| < 1$

$\frac{ar^k}{1-r} = \frac{9/5}{1-3/5} = \frac{\frac{9}{5}}{2/5}$

$\boxed{\frac{9}{5} \cdot \frac{5}{2} = \frac{9}{2}}$ ← converges

3. $\sum_{n=1}^{\infty} \frac{2n}{\sqrt{n^2+1}}$

$\lim_{n\to\infty} a_n = \boxed{2}$

Diverges by the n^{th}-Term Test.

4. $\sum_{n=1}^{\infty} \frac{e^{n+1}}{\pi^n}$

$\frac{e^n \cdot e^1}{\pi^n} = \underbrace{e\left(\frac{e}{\pi}\right)^n}_{\text{Geometric}}$

$|r| < 1$

$\lim_{n\to\infty} a_n = 0$

$\frac{ar^k}{1-r} = \frac{\frac{e^2}{\pi}}{1-e/\pi} \cdot \frac{\pi}{\pi} = \boxed{\frac{e^2}{\pi - e}}$

Converges!

5. $\sum_{n=1}^{\infty} \frac{7^n+1}{7^{n+1}}$

$\frac{7^n}{7^n \cdot 7^1} + \frac{1}{7^n \cdot 7^1}$

$\lim_{n\to\infty} \left(\frac{1}{7} + \frac{1}{7^n \cdot 7}\right) = \boxed{\frac{1}{7}}$

Diverges by the n^{th}-Term Test.

6. $\sum_{n=0}^{\infty} 5\left(\frac{5}{2}\right)^n$

$\lim_{n\to\infty} a_n = \infty$

Diverges by the n^{th}-Term Test.

or

Diverges because geometric series with $r > 1$

7. The nth-Term Test can be used to determine divergence for which of the following series?

I. $\sum_{n=1}^{\infty} \sin 2n$ II. $\sum_{n=1}^{\infty}\left(2+\frac{3}{n}\right)$ III. $\sum_{n=1}^{\infty}\frac{n^3+1}{n^2}$

$\lim_{n\to\infty} a_n =$ Limit does not exist.
Diverges

$\lim_{n\to\infty} a_n = 2$
Diverges

$\lim_{n\to\infty} a_n = \infty$
Diverges

(A) II only (B) III only (C) I and II only (D) I, II, and III

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8. The nth-Term Test can be used to determine divergence for which of the following series?

I. $\sum_{n=1}^{\infty} \ln\left(\frac{n-1}{n}\right)$ II. $\sum_{n=1}^{\infty}\frac{3n-2n^2}{5n^2}$ III. $\sum_{n=1}^{\infty} 3\left(\frac{5}{4}\right)^n$

$\lim_{n\to\infty} a_n = \ln(1) = 0$
Unknown

$\lim_{n\to\infty} a_n = -\frac{2}{5}$
Diverges

$\lim_{n\to\infty} a_n = \infty$
Diverges

(A) II only (B) II and III only (C) I and II only (D) I, II, and III

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9. If $a_n = \cos\left(\frac{\pi}{2n}\right)$ for $n = 1,2,3,\ldots$, which of the following about $\sum_{n=1}^{\infty} a_n$ must be true?

$\lim_{n\to\infty} a_n = \cos(0) = 1 \longrightarrow$ must diverge

(A) The series converges and $\lim_{n\to\infty} a_n = 0$.

(B) The series diverges and $\lim_{n\to\infty} a_n = 0$

(C) The series diverges and $\lim_{n\to\infty} a_n \neq 0$

(D) The series converges and $\lim_{n\to\infty} a_n \neq 0$