Calculus

10.4 Integral Test for Convergence

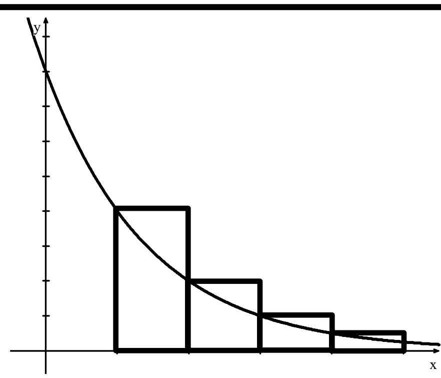
Notes

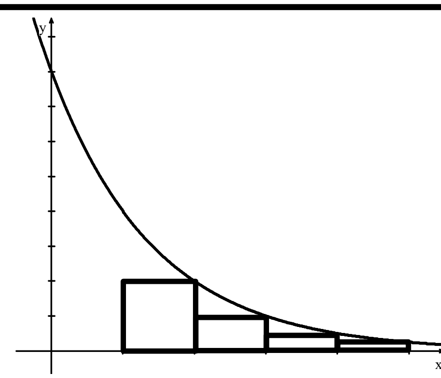
Write your questions and thoughts here!

Integral Test for Convergence

If f is a positive, continuous, and decreasing function for $x \geq k$, and $a_n = f(x)$, then

and





Determine the convergence or divergence of the series

1. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

2. $\sum_{n=1}^{\infty} \frac{1}{n^2}$

10.4 Integral Test for Convergence

Calculus

Practice

If the Integral Test applies, use it to determine whether the series converges or diverges.

1. $\sum_{n=1}^{\infty} \frac{n}{e^n}$

2. $\sum_{n=1}^{\infty} f(n) = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots$

3. $\sum_{n=1}^{\infty} f(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$

4. $\sum_{n=1}^{\infty} \frac{1}{n^2} \sin \frac{\pi}{n}$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

6. $\displaystyle\sum_{n=1}^{\infty} \frac{n^{k-1}}{n^k+2}$, where k is a positive integer. Assume the series meets the criteria for the Integral Test.

7. Let f be a positive, continuous, and decreasing function. If $\int_1^{\infty} f(x)\,dx = 4$, which of the following statements about the series $\displaystyle\sum_{n=1}^{\infty} f(n)$ must be true?

A. $\displaystyle\sum_{n=1}^{\infty} f(n) = 0$

B. $\displaystyle\sum_{n=1}^{\infty} f(n)$ converges, and $\displaystyle\sum_{n=1}^{\infty} f(n) > 4$

C. $\displaystyle\sum_{n=1}^{\infty} f(n)$ converges, and $\displaystyle\sum_{n=1}^{\infty} f(n) < 4$

D. $\displaystyle\sum_{n=1}^{\infty} f(n)$ diverges, and $\displaystyle\sum_{n=1}^{\infty} f(n) = 0$

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8. Explain why the Integral Test does not apply for the series $\displaystyle\sum_{x=1}^{\infty} e^x \sin x$.

9. Show that the series $\displaystyle\sum_{x=1}^{\infty} \frac{\tan^{-1} x}{x^2+1}$ meets the criteria to apply the Integral Test for convergence.

10. Let f be positive, continuous, and decreasing on $[1, \infty)$, such that $a_n = f(n)$. If $\sum_{n=1}^{\infty} a_n = 7$, which of the following must be true?

A. $\lim_{n\to\infty} a_n = 7$

B. $\int_1^{\infty} f(x)\,dx = 7$

C. $\int_1^{\infty} f(x)\,dx$ diverges

D. $\int_1^{\infty} f(x)\,dx$ converges

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11. Which of the following can be used to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{n+3}{n+4}$?

I. Properties of Geometric Series
II. nth-Term Test
III. Integral Test

A. I only

B. II only

C. III only

D. II and III only

E. I, II, and III

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12. Which of the following can be used to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{1}{2^n}$?

I. Properties of Geometric Series
II. nth-Term Test
III. Integral Test

A. I only

B. II only

C. III only

D. I and II only

E. I and III only

10.4 Integral Test for Convergence

Test Prep

13. Consider the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^3}$. The integral test can be used to determine convergence or divergence of the series because $f(x) = \frac{1}{x^3}$ is positive, continuous, and decreasing on $[1, \infty)$. Which of the following is true?

A. $1 + \int_1^{\infty} \frac{1}{x^3}\,dx < \sum_{n=1}^{\infty} \frac{1}{n^3} < \int_1^{\infty} \frac{1}{x^3}\,dx$

B. $\int_1^{\infty} \frac{1}{x^3}\,dx < \sum_{n=1}^{\infty} \frac{1}{n^3} < 1 + \int_1^{\infty} \frac{1}{x^3}\,dx$

C. $\sum_{n=1}^{\infty} \frac{1}{n^3} < \int_1^{\infty} \frac{1}{x^3}\,dx < 1 + \int_1^{\infty} \frac{1}{x^3}\,dx$

D. $\int_1^{\infty} \frac{1}{x^3}\,dx < 1 + \int_1^{\infty} \frac{1}{x^3}\,dx < \sum_{n=1}^{\infty} \frac{1}{n^3}$