Write your questions and thoughts here!

p-Series

Let p be a positive constant of the series $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots$

The series converges if

The series diverges if

Harmonic Series

Do the following series converge or diverge?

$$1. \quad \sum_{n=1}^{\infty} \frac{1}{n^3}$$

$$2. \quad \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$

For what values of k will the series converge?

3.
$$\sum_{n=1}^{\infty} \frac{1}{n^{2k-5}}$$

4.
$$\sum_{n=1}^{\infty} \frac{1}{n(n^{2k})}$$

$$5. \sum_{n=1}^{\infty} \frac{n}{n^{4k} + 5}$$

Things we should now recognize

Series

Tests for convergence/divergence

Geometric

Nth Term Test for Divergence

Harmonic

Integral Test

p-Series

10.5 Harmonic Series and p-series

Determine the convergence or divergence of the following *p*-series.

1.
$$\sum_{n=1}^{\infty} n^{-\frac{3}{2}}$$

$$2. \quad \sum_{n=1}^{\infty} \frac{1}{n^{0.13}}$$

$$3. \sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$$

What are all the values of p for which...

4.
$$\sum_{n=1}^{\infty} \frac{2n}{n^p + 2}$$
 converges?

5.
$$\sum_{n=1}^{\infty} \frac{1}{n^{3p}}$$
 diverges?

6. Both series
$$\sum_{n=1}^{\infty} n^{-5p}$$
 and $\sum_{n=1}^{\infty} \left(\frac{p}{5}\right)^n$ converge?

7.
$$\int_{1}^{\infty} \frac{1}{x^{3p+4}} dx$$
 converges?

Find the positive values of p for which the infinite series converge?

8.
$$\sum_{n=1}^{\infty} \left(\frac{4}{p}\right)^n$$

$$9. \quad \sum_{n=1}^{\infty} \frac{n}{(n^2+1)^p}$$

10.
$$\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$$

- 11. Which of the following infinite series converge?
 - I. $\sum_{n=1}^{\infty} n^{-\frac{1}{2}}$

- II. $\sum_{n=1}^{\infty} \left(\frac{e}{2}\right)^{-n}$
- III. $\sum_{n=1}^{\infty} \frac{1}{n^e}$

A. None

B. II only

C. III only

D. I and II only

- E. II and III only
- 12. Which of the following infinite series converge?

$$I. \qquad \sum_{n=1}^{\infty} 3^{-n}$$

II.
$$\sum_{n=1}^{\infty} \frac{1}{(3n+1)^3}$$

III.
$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$$

A. I only

B. II only

C. III only

D. I and II only

- E. I and III only
- 13. Which of the following infinite series is a divergent *p*-series?
 - A. $\sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n$
- B. $\sum_{n=1}^{\infty} n^{-\frac{1}{2}}$
- C. $\sum_{n=1}^{\infty} n^{-\frac{3}{2}}$
- D. $\sum_{n=1}^{\infty} n^{\frac{3}{2}}$

14. Which of the following is not a p-series?

$$A. \quad \sum_{n=1}^{\infty} n^{-3}$$

B.
$$\sum_{n=1}^{\infty} \frac{1}{n}$$

C.
$$\sum_{n=1}^{25} \frac{1}{n^{\pi}}$$

$$D. \sum_{n=1}^{\infty} \frac{1}{\pi^n}$$

15. Which of the following is a harmonic series?

A.
$$\sum_{n=1}^{\infty} \frac{1}{3n}$$

B.
$$\sum_{n=1}^{\infty} \frac{1}{n}$$

C.
$$\sum_{n=1}^{1000} \frac{1}{n}$$

D.
$$\sum_{n=1}^{\infty} \frac{3n^2}{4n^2 + 1}$$

16. Find the positive values of k for which the series $\sum_{n=3}^{\infty} \frac{1}{(n \ln n)(\ln(\ln n))^k}$ converges.