10.5 Harmonic Series and p-series

Calculus

Solutions

Practice

Determine the convergence or divergence of the following p-series.

1. $\sum_{n=1}^{\infty} n^{-\frac{3}{2}}$ $= \frac{1}{n^{3/2}}$ $p=\frac{3}{2}$

$p>1$

Converges

2. $\sum_{n=1}^{\infty} \frac{1}{n^{0.13}}$ $p=0.13$

$0<p\leq 1$

Diverges

3. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$ $= \frac{1}{n^{3/2}}$ $p=\frac{3}{2}$

$p>1$

converges

What are all the values of p for which…

4. $\sum_{n=1}^{\infty} \frac{2n}{n^p+2}$ converges?

$\frac{n}{n}\cdot\frac{2}{\left(n^{p-1}+\frac{2}{n}\right)}$

To converge, $p-1>1$

$p>2$

5. $\sum_{n=1}^{\infty} \frac{1}{n^{3p}}$ diverges?

Diverges if $3p\leq 1$

$p\leq \frac{1}{3}$

6. Both series $\sum_{n=1}^{\infty} n^{-5p}$ and $\sum_{n=1}^{\infty} \left(\frac{p}{5}\right)^n$ converge?

$\frac{1}{n^{5p}}$

$5p>1$

$p>\frac{1}{5}$

Geometric

$\left|\frac{p}{5}\right|<1$

$\frac{p}{5}<1$

$p<5$

$\frac{1}{5}<p<5$

7. $\int_1^{\infty} \frac{1}{x^{3p+4}}\,dx$ converges?

$3p+4>1$

$3p>-3$

$p>-1$

Find the positive values of p for which the infinite series converge?

8. $\sum_{n=1}^{\infty} \left(\frac{4}{p}\right)^n$

Geometric!

$\frac{4}{p}<1$

$4<p$

$p>4$

9. $\sum_{n=1}^{\infty} \frac{n}{(n^2+1)^p}$

$u=n^2+1$

$\frac{du}{2n}=dn$

$\int_1^{\infty} \frac{1}{2u^p}\,du$

$p>1$

10. $\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$

$2p>1$

$p>\frac{1}{2}$

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Test Prep

11. Which of the following infinite series converge?

I. $\sum_{n=1}^{\infty} n^{-\frac{1}{2}}$ p-series diverges $\frac{1}{n^{\frac{1}{2}}}$ $p=\frac{1}{2}$ $p<1$

II. $\sum_{n=1}^{\infty} \left(\frac{e}{2}\right)^{-n}$ $\left(\frac{2}{e}\right)^{n}$ Geometric converges $r<1$

III. $\sum_{n=1}^{\infty} \frac{1}{n^e}$ $p=e$ $p>1$ p-series converges

A. None
B. II only
C. III only
D. I and II only
E. II and III only

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12. Which of the following infinite series converge?

I. $\sum_{n=1}^{\infty} 3^{-n}$ $\left(\frac{1}{3}\right)^{n}$ Geometric converges $r<1$

II. $\sum_{n=1}^{\infty} \frac{1}{(3n+1)^3}$ $u=3n+1$ $\frac{du}{3}=dn$ $\int_1^{\infty} \frac{1}{3u^3}\,dn$ Integral test or p-series $p>1 \rightarrow$ converges

III. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$ $p=\frac{1}{5}$ $p<1$ p-series diverges

A. I only
B. II only
C. III only
D. I and II only
E. I and III only

$0<p<1$

13. Which of the following infinite series is a divergent p-series?

A. $\sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^{n}$ Geometric

B. $\sum_{n=1}^{\infty} n^{-\frac{1}{2}}$ p-series and $p<1$ $\frac{1}{n^{\frac{1}{2}}}$

C. $\sum_{n=1}^{\infty} n^{-\frac{3}{2}}$ p-series but $p>1$ $\frac{1}{n^{\frac{3}{2}}}$

D. $\sum_{n=1}^{\infty} n^{\frac{3}{2}}$ not p series Int test or n^{th} term test

14. Which of the following is not a p-series?

A. $\sum_{n=1}^{\infty} n^{-3}$ ($\frac{1}{n^3}$)

B. $\sum_{n=1}^{\infty} \frac{1}{n}$

C. $\sum_{n=1}^{25} \frac{1}{n^{\pi}}$

D. $\sum_{n=1}^{\infty} \frac{1}{\pi^n}$ (Geometric ↓)

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15. Which of the following is a harmonic series?

A. $\sum_{n=1}^{\infty} \frac{1}{3n}$

B. $\sum_{n=1}^{\infty} \frac{1}{n}$

C. $\sum_{n=1}^{1000} \frac{1}{n}$

D. $\sum_{n=1}^{\infty} \frac{3n^2}{4n^2+1}$

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16. Find the positive values of k for which the series $\sum_{n=3}^{\infty} \frac{1}{(n \ln n)(\ln(\ln n))^k}$ converges.

$u = \ln(\ln n)$

$du = \frac{1}{\ln n} \cdot \frac{1}{n}\, dn$

$n \ln n \cdot du = dn$

$\int_3^{\infty} \frac{1}{u^k}\, du$

$k > 1$