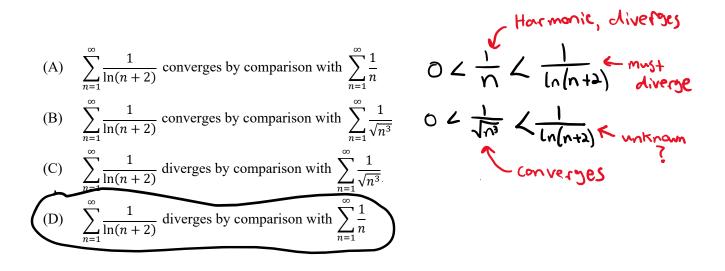
10.6 Comparison Tests for Convergence

Calculus

1. Which of the following statements about convergence of the series $\sum_{n=1}^{\infty} \frac{1}{\ln(n+2)}$ is true?



Solutions

Practice

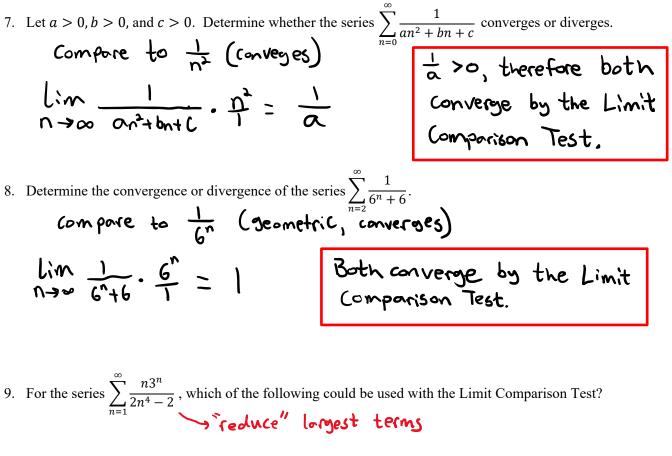
2. Which of the following series converges?

(A)
$$\sum_{n=1}^{\infty} \frac{3n}{n^3 + 2}$$
 compose to $\frac{1}{n^2}$, which converses
Lim $\frac{3n}{n^3 + 2}$ compose to $\frac{1}{n^2}$, which converses
Lim $\frac{3n}{n^3 + 2} \cdot \frac{n^2}{1} = \frac{3n^3}{n^3 + 2} = 3\sqrt{pos} \frac{1}{pinite}$
Both converge
(C) $\sum_{n=1}^{\infty} \frac{7n}{n^2 + 1}$ compose to $\frac{1}{n}$ (Aiverses) (D) $\sum_{n=1}^{\infty} \frac{5^n}{4^n + 1}$ compose to $(\frac{5}{44})^n$
Lim $\frac{7n}{n^2 + 1} \cdot \frac{n}{1} = \frac{7n^2}{n^2 + 1} = 7$
Both diverse
Both diverse
Both diverse
Both diverse
Both diverse

3. Use the Comparison Test to determine whether the series
$$\sum_{n=1}^{\infty} \frac{1}{2+5^n}$$
 converges or diverges.
Compare to $\frac{1}{5^n}$, which converges (geometric)
 $0 < \frac{1}{2+5^n} < \frac{1}{5^n}$
Both converge by
Comparison Test

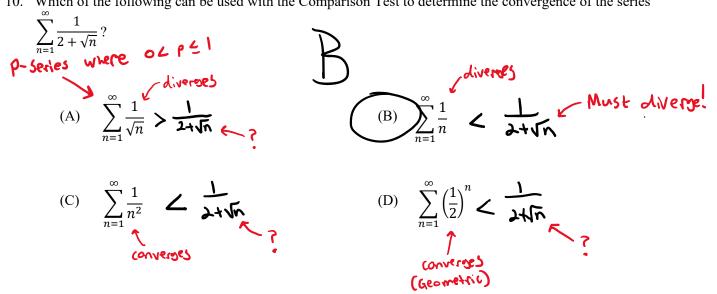
4. Which of the following series can be used with the Limit Comparison Test to determine convergence of the

series
$$\sum_{n=1}^{\infty} \frac{n^3}{n^4 + 3}$$
? **``redu(e**" to $\frac{1}{n}$ for Comparison $\frac{1}{n}$ diverges
(A) $\sum_{n=1}^{\infty} \frac{n}{n+3}$ (B) $\sum_{n=1}^{\infty} \frac{1}{n^2+3}$
(C) $\sum_{n=1}^{\infty} \frac{1}{n}$ (D) $\sum_{n=1}^{\infty} \frac{1}{n^4}$
5. Consider the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$, where $a_n > 0$ and $b_n > 0$ for $n \ge 1$. If $\sum_{n=1}^{\infty} a_n$ diverges which of the following must be true? If $O \subset a_n \le b_n$ then $f \ge b_n$ diverges
(A) If $a_n \le b_n$, then $\sum_{n=1}^{\infty} b_n$ converges. (B) If $a_n \le b_n$, then $\sum_{n=1}^{\infty} b_n$ diverges.
(C) If $b_n \le a_n$, then $\sum_{n=1}^{\infty} b_n$ converges. (D) If $b_n \le a_n$, then $\sum_{n=1}^{\infty} b_n$ diverges.
(C) If $b_n \le a_n$, then $\sum_{n=1}^{\infty} b_n$, where $a_n > 0$ and $b_n > 0$ for $n \ge 1$. If $\sum_{n=1}^{\infty} b_n$ diverges.
(C) If $b_n \le a_n$, then $\sum_{n=1}^{\infty} b_n$, where $a_n > 0$ and $b_n > 0$ for $n \ge 1$. If $\sum_{n=1}^{\infty} b_n$ diverges.
(C) If $b_n \le a_n$, then $\sum_{n=1}^{\infty} b_n$, where $a_n > 0$ and $b_n > 0$ for $n \ge 1$. If $\sum_{n=1}^{\infty} b_n$ diverges.
(C) If $b_n \le a_n$, then $\sum_{n=1}^{\infty} b_n$, where $a_n > 0$ and $b_n > 0$ for $n \ge 1$. If $\sum_{n=1}^{\infty} b_n$ diverges.
(C) If $b_n \le a_n$, then $\sum_{n=1}^{\infty} a_n$ diverges.
(D) If $b_n \le a_n$, then $\sum_{n=1}^{\infty} a_n$ converges.
(A) If $a_n \le b_n$, then $\sum_{n=1}^{\infty} a_n$ diverges.
(B) If $a_n \le b_n$, then $\sum_{n=1}^{\infty} a_n$ converges.
(C) If $b_n \le a_n$, then $\sum_{n=1}^{\infty} a_n$ diverges.
(D) If $b_n \le a_n$, then $\sum_{n=1}^{\infty} a_n$ converges.

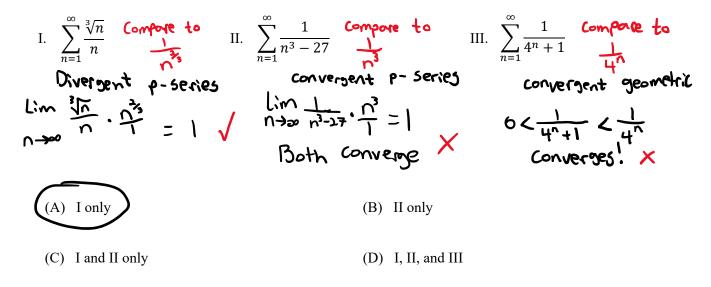




10. Which of the following can be used with the Comparison Test to determine the convergence of the series



11. Which of the following series diverge?



12. Consider the series $\sum_{n=2}^{\infty} \frac{1}{n^p \ln n}$, where $p \ge 0$. For what values of p is the series convergent? Compare to $\frac{1}{n^p}$. For convergence, we need $\frac{1}{n^p}$ to converge, $0 < \frac{1}{n^p \ln(n)} \le \frac{1}{n^p}$. P-Series! $P \ge 1$

13. Determine whether the series $\sum_{n=1}^{\infty} \frac{n-3}{n^3}$ converges or diverges. Compare to $\frac{1}{n^2}$, convergent p-series Lim $\frac{n-3}{n^3} \cdot \frac{n^2}{1} = \lim_{n \to \infty} \frac{n-3}{n} = 1$ Both converge.

14. Consider the series $1 + \frac{1}{5} + \frac{1}{9} + \frac{1}{13} + \dots = \sum_{n=1}^{\infty} \frac{1}{4n-3}$. Use the Limit Comparison Test with the series $\sum_{n=1}^{\infty} \frac{1}{4n}$ to determine the convergence of the series. $\lim_{n \to \infty} \frac{1}{4n-3} \cdot \frac{4n}{1} = \lim_{n \to \infty} \frac{4n}{4n-3} = 1$ Divergent
(Hormonic p-series)
Bath diverge.

10.6 Comparison Tests for Convergence

Test Prep

15. Consider the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$, where $a_n > 0$ and $b_n > 0$ for $n \ge 1$. If $a_n \le b_n$, then which of the following must be true? (A) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} b_n$ diverges. (B) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} b_n$ converges. (C) If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges. (D) If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ converges.

16. Consider the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$, where $a_n > 0$ and $b_n > 0$ for $n \ge 1$. If $\lim_{n \to \infty} \frac{a_n}{b_n} = 2$, then which of the following must be true?

- I. If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} b_n$ diverges. II. If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ converges. III. If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} b_n$ converges. IV. If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.
- (A) I only
 (B) II only
 (C) III only
 (D) IV only
 (E) I and II only
 (F) III and IV only