Calculus

10.7 Alternating Series Test

Notes

Write your questions and thoughts here!

Alternating Series Test

If $a_n > 0$, then the alternating series $\sum_{n=1}^{\infty}(-1)^n a_n$ and $\sum_{n=1}^{\infty}(-1)^{n+1} a_n$ converge if **BOTH** of the following conditions are met:

1.

2.

Ways to check if a_n is decreasing.

- Take the 1st derivative and see if it is negative.
- Usually, it is obvious.
- Could manipulate $a_{n+1} \leq a_n$

Determine if the following series converge or diverge.

1. $\sum_{n=1}^{\infty}(-1)^n \frac{1}{n}$

2. $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{n+5}{(n+2)(n+3)}$

3. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(n+7)}{n}$

4. $\sum_{n=1}^{\infty}\cos(n\pi)\frac{1}{n}$

Write your questions and thoughts here!

5. The following is not an alternating series. Look carefully to see if you can tell why not.

$$\sum_{n=1}^{\infty} \frac{(-1)^n \cos(n\pi)\, n}{n^2+1}$$

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Calculus

Practice

1. Explain why the Alternating Series Test does not apply to the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+1)}{n}$.

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2. The Alternating Series Test can be used to show convergence of which of the following alternating series?

I. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ II. $\sum_{n=2}^{\infty} (-1)^{n+1}\left(\frac{n}{n^2+4}\right)$ III. $\sum_{n=1}^{\infty} (-1)^{n}\left(\frac{4n}{5n+3}\right)$

A. I only B. II only C. III only

D. I and II only E. I, II, and III

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3. Which of the following series converge?

A. $\sum_{n=1}^{\infty} (-1)^n \left(\frac{1-2n}{n}\right)$

B. $\sum_{n=1}^{\infty} (-1)^n \left(\frac{n+1}{3n}\right)$

C. $\sum_{n=1}^{\infty} (-1)^n \left(\frac{n^3}{2\sqrt{n}}\right)$

D. $\sum_{n=1}^{\infty} (-1)^n \left(\frac{2\sqrt{n}}{n^3}\right)$

Use the Alternating Series Test to show the series are convergent.

4. $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{1}{n^2}\right)$

5. $\sum_{n=1}^{\infty}(-1)^{n}\left(\frac{1}{3^n}\right)$

6. **Calculator active.** Which of the following statements are true about the series $\sum_{n=2}^{\infty} a_n$, where $a_n = \frac{(-1)^n}{(-1)^n+\sqrt{n}}$

I. The series is alternating.
II. $|a_{n+1}| \leq |a_n|$ for $n \geq 2$.
III. $\lim_{n\to\infty} a_n = 0$

A. I only B. I and II only C. I and III only D. I, II, and III

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7. **Calculator active.** Which of the following statements about the series $\sum_{n=1}^{\infty}(-1)^{n+1}a_n$, where $a_n = \frac{2+\cos n}{n^2}$ is true?

A. The series converges by the Alternating Series Test

B. The Alternating Series Test cannot be used because the series is not alternating.

C. The Alternating Series Test cannot be used because $\lim_{n\to\infty} a_n \neq 0$.

D. The Alternating Series Test cannot be used because the terms of a_n are not decreasing.

8. The Alternating Series Test can be used to show convergence for which of the following series?

A. $\frac{2}{1}-\frac{3}{2}+\frac{4}{3}-\frac{5}{4}+\frac{6}{5}-\cdots$, where $a_n=\frac{(-1)^{n+1}(n+1)}{n}$.

B. $\frac{2}{1}-\frac{1}{1}+\frac{2}{2}-\frac{1}{2}+\frac{2}{3}-\frac{1}{3}+\frac{2}{4}-\frac{1}{4}+\cdots$

C. $1-\frac{1}{4}+\frac{1}{9}-\frac{1}{16}+\frac{1}{25}-\frac{1}{36}+\cdots$, where $a_n=(-1)^{n+1}\frac{1}{n^2}$

D. $\frac{3}{2}-\frac{2}{2}+\frac{3}{3}-\frac{2}{3}+\frac{3}{4}-\frac{2}{4}+\cdots$

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9. For which of the following series can the Alternating Series Test not be used?

A. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^5}$

B. $\sum_{n=2}^{\infty}\frac{(-1)^n\ln(n^3)}{n}$

C. $\sum_{n=4}^{\infty}\frac{(-1)^n n}{n-3}$

D. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$

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10. Which of the following statements about the series $\sum_{n=1}^{\infty}\frac{(-1)^n}{n!}$ is true?

A. The series diverges by comparison to $\frac{1}{n}$.

B. The series converges by comparison to $\frac{1}{n}$.

C. The series diverges by the Alternating Series Test.

D. The series converges by the Alternating Series Test.

11. Which of the following statements are true about the series $\sum_{n=1}^{\infty} \frac{(-1)^n(n+1)!}{(n)!}$?

I. The series is alternating.

II. $|a_{n+1}| \leq |a_n|$ for $n \geq 1$.

III. $\lim_{n\to\infty} a_n = 0$

A. I only B. I and II only C. I and III only D. I, II, and III

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Test Prep

12. The Alternating Series Test can be used to show convergence for which of the following series?

I. $\sum_{n=1}^{\infty} (-1)^{n+1}\left(\frac{1}{n^2}\right)$

II. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sin n}{n^2}$

III. $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{2}+1} - \frac{1}{\sqrt{2}-1} + \frac{1}{\sqrt{3}+1} - \frac{1}{\sqrt{3}-1} + \frac{1}{\sqrt{4}+1} - \frac{1}{\sqrt{4}-1} + \cdots\right)$

A. I only B. I and II only C. II and III only D. I, II, and III

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13. If $\sum_{n=1}^{\infty} \frac{(-1)^n}{a_n}$ converges, which of the following must be true?

A. $\lim_{n\to\infty} a_n = 0$ and $a_{n+1} \geq a_n > 0$ for $n \geq 1$.

B. $\lim_{n\to\infty} a_n = \infty$ and $a_{n+1} \leq a_n$ for $n \geq 1$.

C. $\lim_{n\to\infty} a_n = 0$ and $a_{n+1} \leq a_n$ for $n \geq 1$.

D. $\lim_{n\to\infty} a_n = \infty$ and $a_{n+1} \geq a_n > 0$ for $n \geq 1$.

14. For what value of $k > 0$ will both $\sum_{n=1}^{\infty} \frac{(-1)^{kn}}{n}$ and $\sum_{n=1}^{\infty} \left(\frac{6}{k}\right)^n$ diverge?

A. 3 B. 4 C. 5 D. 7