1. Explain why the Alternating Series Test does not apply to the series $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(n+1)}{n}$

but
$$\lim_{n\to\infty} \frac{n+1}{n} = 1 \neq 0$$

2. The Alternating Series Test can be used to show convergence of which of the following alternating series?

$$I. \quad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$$

II.
$$\sum_{n=2}^{\infty} (-1)^{n+1} \left(\frac{n}{n^2 + 4} \right)^{n+1}$$

III.
$$\sum_{n=1}^{\infty} (-1)^n \left(\frac{4n}{5n+3} \right)$$

$$\int_{n\to\infty}^{n=1} \ln z = 0$$

I.
$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$$
II.
$$\sum_{n=2}^{\infty} (-1)^{n+1} \left(\frac{n}{n^2+4}\right)$$
III.
$$\sum_{n=1}^{\infty} (-1)^n \left(\frac{4n}{5n+3}\right)$$

$$\downarrow \lim_{n \to \infty} \frac{1}{n} = 0$$

$$\downarrow \lim_{n \to \infty} \frac{1}{5n+3} = 0$$

$$\downarrow \lim_{n \to \infty} \frac{1}{5n+3} = 0$$

$$\sqrt{\frac{1}{n}}$$
 is decreosing $\sqrt{\frac{2}{2^{2}+4}} > \frac{3}{3^{2}+4} > \frac{4}{4^{2}+4} > \cdots$

A. I only

B. II only

C. III only

- D. I and II only
- E. I, II, and III
- 3. Which of the following series converge?

$$XA.$$
 $\sum_{n=1}^{\infty} (-1)^n \left(\frac{1-2n}{n}\right)$ $\lim_{n\to\infty} \alpha_n = -2$

$$\lambda$$
 B. $\sum_{n=1}^{\infty} (-1)^n \left(\frac{n+1}{3n}\right)$ $\lim_{n\to\infty} a_n = \frac{1}{3}$

$$X \text{ C.} \quad \sum_{n=1}^{\infty} (-1)^n \left(\frac{n^3}{2\sqrt{n}}\right)$$
 $\sum_{n=1}^{\infty} (-1)^n \left(\frac{n^3}{2\sqrt{n}}\right)$

$$\underbrace{
\begin{array}{c}
\sum_{n=1}^{\infty} (-1)^n \left(\frac{2\sqrt{n}}{n^3} \right)
\end{array}}_{n=1}$$

Use the Alternating Series Test to show the series are convergent.

$$4. \quad \sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{1}{n^2}\right)$$

$$\frac{1}{1^2} > \frac{1}{2^2} > \frac{1}{3^2} \cdots \sqrt{\text{decreasing}}$$

Converges by the Alternating Series Test

5.
$$\sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{3^n}\right)$$

$$\lim_{n \to \infty} \left(\frac{1}{3^n}\right) = 0 \quad \checkmark$$

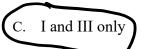
$$\frac{1}{3^{1}} > \frac{1}{3^{2}} > \frac{1}{3^{3}} \sim ... \checkmark$$

Converges by the Alternating Series Test.

- 6. Calculator active. Which of the following statements are true about the series $\sum a_n$, where $a_n = \frac{(-1)^n}{(-1)^n + \sqrt{n}}$
 - I. The series is alternating.
 - II. $|a_{n+1}| \le |a_n|$ for $n \ge 2$. III. $\lim_{n \to \infty} a_n = 0$

	n=	n=2	
1	an	Sign	
J	1+V2 × 0.414	იავ	
3	<u>-1</u> -1+√3 × -1.366	neg	
4	1+14 \$ 0.333	pos	
5	<u>-1</u> +√5 ≈-0.809	neg	

- A. I only
- B. I and II only



- D. I, II, and III
- 7. Calculator active. Which of the following statements about the series $\sum_{n=0}^{\infty} (-1)^{n+1} a_n$, where $a_n = \frac{2 + \cos n}{n^2}$ is lim an = 0 V

an is always positive, so the series is alternating. a - 0.396

- A. The series converges by the Alternating Series Test
- B. The Alternating Series Test cannot be used because the series is not alternating.
- C. The Alternating Series Test cannot be used because $\lim_{n\to\infty} a_n \neq 0$.
- 102 × 0.112
- \a_3/\$0.0841
- D. The Alternating Series Test cannot be used because the terms of a_n are not decreasing.

8. The Alternating Series Test can be used to show convergence for which of the following series?

$$X A. \frac{2}{1} - \frac{3}{2} + \frac{4}{3} - \frac{5}{4} + \frac{6}{5} - \dots$$
, where $a_n = \frac{(-1)^{n+1}(n+1)}{n}$.

X B.
$$\frac{2}{1} - \frac{1}{1} + \frac{2}{2} - \frac{1}{2} + \frac{2}{3} - \frac{1}{3} + \frac{2}{4} - \frac{1}{4} + \cdots$$
 Not decreasing

C.
$$1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} - \frac{1}{36} + \cdots$$
, where $a_n = (-1)^{n+1} \frac{1}{n^2}$ \(\text{lim} \alpha_n = 0\), \(\alpha_{n+1}\) \(\alpha_n\)

$$\times$$
 D. $\frac{3}{2} - \frac{2}{2} + \frac{3}{3} - \frac{2}{3} + \frac{3}{4} - \frac{2}{4} + \cdots$ Not decreasing

9. For which of the following series can the Alternating Series Test not be used?

$$A. \quad \sum_{n=1}^{\infty} \frac{(-1)^n}{n^5}$$

B.
$$\sum_{n=2}^{\infty} \frac{(-1)^n \ln(n^3)}{n}$$

$$\left(\begin{array}{ccc}
\sum_{n=4}^{\infty} \frac{(-1)^n n}{n-3} & \longrightarrow & \lim_{n \to \infty} \frac{n}{n-3} & = \end{array}\right) \neq 0$$

$$D. \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

10. Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$ is true?

$$\frac{\alpha_n = \frac{1}{n!}}{\lim_{n \to \infty} \alpha_n = 0}$$

$$\left|a_{n+1}\right| < \left|a_{n}\right|$$

- A. The series diverges by comparison to $\frac{1}{n}$.
- B. The series converges by comparison to $\frac{1}{n}$.
- C. The series diverges by the Alternating Series Test.
- D. The series converges by the Alternating Series Test.

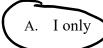
11. Which of the following statements are true about the series
$$\sum_{n=0}^{\infty} \frac{(-1)^n (n+1)!}{(n)!}$$
?

II.
$$|a_{n+1}| \le |a_n|$$
 for $n \ge 1$.

III.
$$\lim_{n\to\infty} a_n = 0$$

$$\lim_{n\to\infty}\frac{(n+1)!}{n!}=\infty$$

$$-\frac{3!}{1!}+\frac{3!}{3!}-\frac{3!}{4!}$$



- B. I and II only
- C. I and III only
- D. I, II, and III

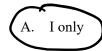
10.7 Alternating Series Test

Test Prep

12. The Alternating Series Test can be used to show convergence for which of the following series?

$$\int_{n=1}^{\infty} (-1)^{n+1} \left(\frac{1}{n^2}\right) \qquad \lim_{n \to \infty} \alpha_n = 0 \quad \text{and} \quad |\alpha_{n+1}| < |\alpha_n|$$

$$X$$
 II.
$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sin n}{n^2}$$
 not decreasing.



- B. I and II only
- C. II and III only
- D. I, II, and III

13. If
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{a_n}$$
 converges, which of the following must be true?

Small fractions

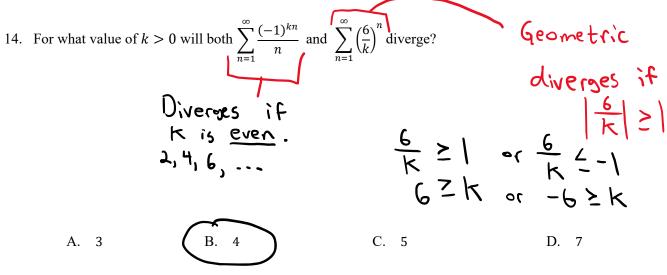
V5. by fractions

A. $\lim_{n\to\infty} a_n = 0$ and $a_{n+1} \ge a_n > 0$ for $n \ge 1$.

B. $\lim_{n\to\infty} a_n = \infty$ and $a_{n+1} \le a_n$ for $n \ge 1$.

C. $\lim_{n\to\infty} a_n = 0$ and $a_{n+1} \le a_n$ for $n \ge 1$.

D.
$$\lim_{n\to\infty} a_n = \infty$$
 and $a_{n+1} \ge a_n > 0$ for $n \ge 1$.



even and less than 6