Solutions

Practice

1. Explain why the Alternating Series Test does not apply to the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+1)}{n}$.

condition 1 states $\lim_{n\to\infty} a_n = 0$ to use the A.S.T.,

but $\lim_{n\to\infty} \frac{n+1}{n} = 1 \neq 0$

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2. The Alternating Series Test can be used to show convergence of which of the following alternating series?

I. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ II. $\sum_{n=2}^{\infty} (-1)^{n+1}\left(\frac{n}{n^2+4}\right)$ III. $\sum_{n=1}^{\infty} (-1)^{n}\left(\frac{4n}{5n+3}\right)$

✓ $\lim_{n\to\infty} \frac{1}{n} = 0$

✓ $\frac{1}{n}$ is decreasing for $n > 1$

✓ $\lim_{n\to\infty} \frac{n}{n^2+4} = 0$

✓ $\frac{2}{2^2+4} > \frac{3}{3^2+4} > \frac{4}{4^2+4} > \ldots$

✗ $\lim_{n\to\infty} \frac{4n}{5n+3} = \frac{4}{5}$

A. I only B. II only C. III only

D. I and II only E. I, II, and III

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3. Which of the following series converge?

✗ A. $\sum_{n=1}^{\infty} (-1)^{n}\left(\frac{1-2n}{n}\right)$ $\lim_{n\to\infty} a_n = -2$

✗ B. $\sum_{n=1}^{\infty} (-1)^{n}\left(\frac{n+1}{3n}\right)$ $\lim_{n\to\infty} a_n = \frac{1}{3}$

✗ C. $\sum_{n=1}^{\infty} (-1)^{n}\left(\frac{n^3}{2\sqrt{n}}\right)$ $\lim_{n\to\infty} a_n = \infty$

D. $\sum_{n=1}^{\infty} (-1)^{n}\left(\frac{2\sqrt{n}}{n^3}\right)$ $\lim_{n\to\infty} a_n = 0$ and $a_{n+1} < a_n$ $\frac{2\sqrt{1}}{1} > \frac{2\sqrt{2}}{8} > \frac{2\sqrt{3}}{27} \cdots$

Use the Alternating Series Test to show the series are convergent.

4. $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{1}{n^2}\right)$

$\lim_{n\to\infty} \frac{1}{n^2} = 0$ ✓

$\frac{1}{1^2} > \frac{1}{2^2} > \frac{1}{3^2} \cdots$ ✓ decreasing

Converges by the Alternating Series Test.

5. $\sum_{n=1}^{\infty}(-1)^{n}\left(\frac{1}{3^n}\right)$

$\lim_{n\to\infty} \left(\frac{1}{3^n}\right) = 0$ ✓

$\frac{1}{3^1} > \frac{1}{3^2} > \frac{1}{3^3} \cdots$ ✓

Converges by the Alternating Series Test.

6. **Calculator active.** Which of the following statements are true about the series $\sum_{n=2}^{\infty} a_n$, where $a_n = \frac{(-1)^n}{(-1)^n+\sqrt{n}}$

I. The series is alternating. ✓
II. $|a_{n+1}| \le |a_n|$ for $n \ge 2$. ✗
III. $\lim_{n\to\infty} a_n = 0$ ✓

n	a_n	Sign
2	$\frac{1}{1+\sqrt{2}} \approx 0.414$	pos
3	$\frac{-1}{-1+\sqrt{3}} \approx -1.366$	neg
4	$\frac{1}{1+\sqrt{4}} \approx 0.333$	pos
5	$\frac{-1}{-1+\sqrt{5}} \approx -0.809$	neg

A. I only B. I and II only C. I and III only D. I, II, and III

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7. **Calculator active.** Which of the following statements about the series $\sum_{n=1}^{\infty}(-1)^{n+1}a_n$, where $a_n = \frac{2+\cos n}{n^2}$ is true?

$\lim_{n\to\infty} a_n = 0$ ✓

a_n is always positive, so the series is alternating. ✓

$|a_1| \approx 0.396$
$|a_2| \approx 0.112$
$|a_3| \approx 0.0841$
$|a_4| \approx 0.0913$

A. The series converges by the Alternating Series Test

B. The Alternating Series Test cannot be used because the series is not alternating.

C. The Alternating Series Test cannot be used because $\lim_{n\to\infty} a_n \neq 0$.

D. The Alternating Series Test cannot be used because the terms of a_n are not decreasing.

8. The Alternating Series Test can be used to show convergence for which of the following series?

X A. $\frac{2}{1}-\frac{3}{2}+\frac{4}{3}-\frac{5}{4}+\frac{6}{5}-\cdots$, where $a_n=\frac{(-1)^{n+1}(n+1)}{n}$. $\lim_{n\to\infty} a_n = 1 \neq 0$

X B. $\frac{2}{1}-\frac{1}{1}+\frac{2}{2}-\frac{1}{2}+\frac{2}{3}-\frac{1}{3}+\frac{2}{4}-\frac{1}{4}+\cdots$ Not decreasing

C. $1-\frac{1}{4}+\frac{1}{9}-\frac{1}{16}+\frac{1}{25}-\frac{1}{36}+\cdots$, where $a_n=(-1)^{n+1}\frac{1}{n^2}$ $\lim_{n\to\infty} a_n = 0$, $|a_{n+1}| < |a_n|$

X D. $\frac{3}{2}-\frac{2}{2}+\frac{3}{3}-\frac{2}{3}+\frac{3}{4}-\frac{2}{4}+\cdots$ not decreasing

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9. For which of the following series can the Alternating Series Test not be used?

A. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^5}$

B. $\sum_{n=2}^{\infty}\frac{(-1)^n\ln(n^3)}{n}$

C. $\sum_{n=4}^{\infty}\frac{(-1)^n n}{n-3}$ $\rightarrow \lim_{n\to\infty}\frac{n}{n-3} = 1 \neq 0$

D. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$

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10. Which of the following statements about the series $\sum_{n=1}^{\infty}\frac{(-1)^n}{n!}$ is true?

$a_n = \frac{1}{n!}$ $|a_{n+1}| < |a_n|$

$\lim_{n\to\infty} a_n = 0$

A. The series diverges by comparison to $\frac{1}{n}$.

B. The series converges by comparison to $\frac{1}{n}$.

C. The series diverges by the Alternating Series Test.

D. The series converges by the Alternating Series Test.

11. Which of the following statements are true about the series $\sum_{n=1}^{\infty} \frac{(-1)^n (n+1)!}{(n)!}$?

I. The series is alternating. ✓

II. $|a_{n+1}| \leq |a_n|$ for $n \geq 1$. ✗

III. $\lim_{n\to\infty} a_n = 0$ ✗

$$\lim_{n\to\infty} \frac{(n+1)!}{n!} = \infty$$

$$-\frac{2!}{1!} + \frac{3!}{2!} - \frac{4!}{3!}$$

A. I only   B. I and II only   C. I and III only   D. I, II, and III

10.7 Alternating Series Test

Test Prep

12. The Alternating Series Test can be used to show convergence for which of the following series?

✓ I. $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{1}{n^2}\right)$ $\lim_{n\to\infty} a_n = 0$ and $|a_{n+1}| < |a_n|$

✗ II. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sin n}{n^2}$ not decreasing!

✗ III. $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{2}+1} - \frac{1}{\sqrt{2}-1} + \frac{1}{\sqrt{3}+1} - \frac{1}{\sqrt{3}-1} + \frac{1}{\sqrt{4}+1} - \frac{1}{\sqrt{4}-1} + \cdots\right)$

not decreasing

A. I only   B. I and II only   C. II and III only   D. I, II, and III

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13. If $\sum_{n=1}^{\infty} \frac{(-1)^n}{a_n}$ converges, which of the following must be true?

* Think about small fractions vs. big fractions

$$0 < \frac{1}{a^{n+1}} \leq \frac{1}{a_n} \longrightarrow \text{true only if } a^{n+1} \geq a^n$$

A. $\lim_{n\to\infty} a_n = 0$ and $a_{n+1} \geq a_n > 0$ for $n \geq 1$.

B. $\lim_{n\to\infty} a_n = \infty$ and $a_{n+1} \leq a_n$ for $n \geq 1$.

C. $\lim_{n\to\infty} a_n = 0$ and $a_{n+1} \leq a_n$ for $n \geq 1$.

D. $\lim_{n\to\infty} a_n = \infty$ and $a_{n+1} \geq a_n > 0$ for $n \geq 1$.

$\lim_{n\to\infty} \frac{1}{a_n}$ must $= 0$. This only happens if $\lim_{n\to\infty} a_n = \infty$

14. For what value of $k > 0$ will both $\sum_{n=1}^{\infty} \frac{(-1)^{kn}}{n}$ and $\sum_{n=1}^{\infty} \left(\frac{6}{k}\right)^n$ diverge?

Geometric

diverges if $\left|\frac{6}{k}\right| \geq 1$

Diverges if k is <u>even</u>.
2, 4, 6, ...

$\frac{6}{k} \geq 1$ or $\frac{6}{k} \leq -1$

$6 \geq k$ or $-6 \geq k$

A. 3

B. 4

C. 5

D. 7

even and less than 6