10.8 Ratio Test

Recall:

$$\frac{(n+1)!}{n!} =$$

$$\frac{3^{n+1}}{3^n} =$$

Ratio Test for Convergence

If $\sum\limits_{n=1}^{\infty} a_n$ has positive terms and…

- $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$, then the series

- $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} > 1$, then the series

- $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$, then

Let's look at two series we already know.

1. $\sum\limits_{n=1}^{\infty} \frac{1}{n}$

2. $\sum\limits_{n=1}^{\infty} \frac{1}{n^2}$

Using the Ratio Test to find convergence or divergence.

3. $\sum\limits_{n=1}^{\infty} \frac{n^2 \cdot 3^{n+1}}{5^n}$

4. $\sum\limits_{n=1}^{\infty} \frac{4^n}{n!}$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{n^5}$

10.8 Ratio Test
Calculus

Practice

Determine whether the following series converges or diverges.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)3^n}{n!}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{5^n}$

3. What are values of $x > 0$ for which the series
$\displaystyle\sum_{n=1}^{\infty} \frac{n6^n}{x^n}$ converges?

4. What are all positive values of p for which the
series $\displaystyle\sum_{n=1}^{\infty} \frac{n^p}{7^n}$ will converge?

   A. $p > 0$ B. $0 < p < 7$

   C. $p > 1$ D. There are no positive
   values where the
   series will converge.

5. Which of the following series converge?

   I. $\displaystyle\sum_{n=1}^{\infty} \frac{7^n}{n!}$

   II. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^{20}}$

   III. $\displaystyle\sum_{n=1}^{\infty} \frac{\pi^{-2n}}{n}$

   A. I only   B. I and II only   C. III only   D. I and III only   E. I, II, and III

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6. If the Ratio Test is applied to the series $\displaystyle\sum_{n=1}^{\infty} \frac{n\pi^n}{15^n}$, which of the following inequalities results, implying that the series converges?

   A. $\displaystyle\lim_{n\to\infty} \frac{n\pi^n}{15^n} < 1$ B. $\displaystyle\lim_{n\to\infty} \frac{15^n}{n\pi^n} < 1$ C. $\displaystyle\lim_{n\to\infty} \frac{(n+1)\pi^{n+1}}{15^{n+1}} < 1$ D. $\displaystyle\lim_{n\to\infty} \frac{(n+1)\pi}{15n} < 1$

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7. If $a_n > 0$ for all n and $\displaystyle\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 6$, which of the following series converges?

   A. $\displaystyle\sum_{n=1}^{\infty} a_n$ B. $\displaystyle\sum_{n=1}^{\infty} \frac{a_n}{n^7}$ C. $\displaystyle\sum_{n=1}^{\infty} \frac{a_n}{7^n}$ D. $\displaystyle\sum_{n=1}^{\infty} \frac{(a_n)^2}{7^n}$

8. Consider the series $\sum_{n=1}^{\infty} \frac{n!}{3^n}$. If the Ratio Test is applied to the series, which of the following inequalities results, implying the series diverges?

A. $\lim_{n\to\infty} \frac{n!}{3^n} < 1$
B. $\lim_{n\to\infty} \frac{n!}{3^n} > 1$
C. $\lim_{n\to\infty} \frac{n+1}{3} < 1$
D. $\lim_{n\to\infty} \frac{n+1}{3} > 1$

9. For which of the series is the Ratio Test inconclusive?

I. $\sum_{n=1}^{\infty} \frac{1}{3n}$

II. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n+1}$

III. $\sum_{n=1}^{\infty} \frac{e^n}{n!}$

A. I only   B. II only   C. I and III only   D. I and II only   E. I, II, and III

10. Apply any appropriate test to determine which of the following series diverges.

I. $\sum_{n=1}^{\infty} \frac{n}{2n^2+1}$

II. $\sum_{n=1}^{\infty} \frac{n!}{9^n}$

III. $\sum_{n=1}^{\infty} \frac{n+1}{4n+1}$

A. I only   B. II only   C. III only   D. I and II only   E. I, II, and III

Match the test for convergence of an infinite series with the conditions of convergence.	
Convergence Test	**Condition of convergence**

11. _____ nth-Term Test

12. _____ Geometric Series

13. _____ p-series

14. _____ Alternating Series Test

15. _____ Integral Test

16. _____ Ratio Test

17. _____ Comparison Test

18. _____ Limit Comparison Test

A. $p > 1$

B. $\lim\limits_{n \to \infty} \dfrac{a_{n+1}}{a_n} < 1$

C. $0 < a_n \le b_n$ and $\sum\limits_{n=1}^{\infty} b_n$ converges

D. $\lim\limits_{n \to \infty} \dfrac{a_n}{b_n} = L > 0$ and $\sum\limits_{n=1}^{\infty} b_n$ converges

E. $|r| < 1$

F. Inconclusive for convergence

G. $|a_{n+1}| \le |a_n|$ and $\lim\limits_{n \to \infty} a_n = 0$

H. $\int_1^{\infty} f(x)\, dx$ converges.

10.8 Ratio Test

Test Prep

19. If the Ratio Test is applied to the series $\sum\limits_{n=1}^{\infty} \dfrac{7^n}{(n+1)!}$, which of the following limits results, implying that the series converges?

A. $\lim\limits_{n \to \infty} \dfrac{7^n}{(n+1)!}$

B. $\lim\limits_{n \to \infty} \dfrac{7}{n+2}$

C. $\lim\limits_{n \to \infty} \dfrac{(n+1)!}{7^n}$

D. $\lim\limits_{n \to \infty} \dfrac{n+2}{7}$

20. Use the Ratio Test to determine the convergence or divergence of the series $\sum\limits_{n=1}^{\infty} \dfrac{n^n}{n!}$.