Solutions

Practice

1. Which of the following series are conditionally convergent?

$\times$ I. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}$ ✓ II. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ ✓ III. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$

Abs: $\sum_{n=1}^{\infty} \frac{1}{n^4}$ converges Absolutely

p-series

Abs: $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges (p-series)

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges (alternating harmonic)

Abs: $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$ diverges (p-series)

$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$

$\lim_{n\to\infty} a_n = 0$ ✓

$\frac{a_{n+1}}{a_n} = \frac{1}{(n+1)^{1/3}} \cdot \frac{n^{1/3}}{1} < 1$ ✓

converges

D

A. I only B. I and II only C. I and III only D. II and III only

Determine whether the series converges absolutely, converges conditionally, or diverges.

2. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n^2+8)}{\pi^n}$

Absolute: $\sum_{n=1}^{\infty} \frac{n^2+8}{\pi^n}$

Ratio Test

$\lim_{n\to\infty} \frac{(n+1)^2+8}{\pi^{n+1}} \cdot \frac{\pi^n}{n^2+8}$

$\lim_{n\to\infty} \frac{(n+1)^2+8}{\pi(n^2+8)}$

$\frac{1}{\pi} < 1$

Converges absolutely

3. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$

Absolute: $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges

Alt. Series Test

$\lim_{n\to\infty} \frac{1}{n} = 0$ ✓

$\frac{1}{n}$ is decreasing ✓

converges conditionally

4. $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{(n+1)^2}$

Abs: $\sum_{n=1}^{\infty} \frac{n^2}{(n+1)^2}$

N^{th} term test

$\lim_{n\to\infty} a_n = 1 \neq 0$

Diverges

Alt. series test

Same

Diverges

5. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{\frac{5}{2}}}$

Abs $\sum_{n=1}^{\infty} \frac{1}{n^{5/2}}$

Converges (p-series, p>1)

Converges Absolutely

6. For which values x is the series $\sum_{n=1}^{\infty} \frac{nx^n}{4^n(n^2+1)}$ conditionally convergent?

$\lim_{n\to\infty} \left| \frac{(n+1)x^{n+1}}{4^{n+1}[(n+1)^2+1]} \cdot \frac{4^n(n^2+1)}{nx^n} \right| < 1$

$\lim_{n\to\infty} \left| \frac{(n+1)\cancel{x^n}\cdot x^1 \cdot \cancel{4^n}(n^2+1)}{\cancel{4^n}\cdot 4^1 \cdot n \cdot \cancel{x^n}\cdot[(n+1)^2+1]} \right| < 1$

$\left|\frac{x}{4}\right| < 1$

$-1 < \frac{x}{4} < 1$

$-4 < x < 4$ ← abs. conv.

At $x=-4$ converges by Alt. Series Test

A. $x = 4$ B. $x = -4$ C. $x > 4$ D. $-4 < x < 4$

7. Which of the following statements is true about the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{1/3}}$.

Abs $\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$ diverges (p-Series)

Alt. series test

$\lim_{n\to\infty} \frac{1}{n^{1/3}} = 0$ ✓

$\frac{1}{n^{1/3}}$ is decreasing ✓

converges

A. The series converges conditionally.

B. The series converges absolutely.

C. The series converges but neither conditionally nor absolutely.

D. The series diverges.

---

8. Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{1+n^{3/2}}$ is true?

Abs: $\sum_{n=1}^{\infty} \frac{1}{1+n^{3/2}}$

compare to $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

converges, $p>1$

$0 < \frac{1}{1+n^{3/2}} < \frac{1}{n^{3/2}}$

Converges!

A. The series converges conditionally.

B. The series converges absolutely.

C. The series converges but neither conditionally nor absolutely.

D. The series diverges.

---

9. Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n}$ is true?

$a_n = \frac{\ln(n)}{n}$

I. Converges Absolutely

II. Diverges

III. Converges Conditionally

Absolute:

Comparison test

$\frac{\ln(n)}{n} > \frac{1}{n}$

$\frac{1}{n}$ diverges (harmonic)

$\frac{\ln(n)}{n}$ also diverges

Alt. Series Test

① $\lim_{n\to\infty} a_n = 0$ ✓

② Decreasing?

$a'_{(n)} = \frac{\frac{1}{n}\cdot n - \ln(n)}{n^2} = 0$

$1 - \ln(n) = 0$

$n = e$ ✓

Decreasing for $n > e$. ✓

Converges

A. I only

B. II only

C. III only

D. I and III only

---

10. For what values of x is the series $\sum_{n=1}^{\infty} \frac{n(x+5)^n}{7^n}$ absolutely convergent?

$\lim_{n\to\infty} \left| \frac{(n+1)(x+5)^{n+1}}{7^{n+1}} \cdot \frac{7^n}{n(x+5)^n} \right| < 1$

$\lim_{n\to\infty} \left| \frac{(n+1)}{n} \cdot \frac{(x+5)}{7} \right| < 1$

$\left| \frac{x+5}{7} \right| < 1$

$-1 < \frac{x+5}{7} < 1$

$-7 < x+5 < 7$

$-12 < x < 2$ ← abs conv.

A. $x = -12$

B. $x = 2$

C. $x > 2$

D. $-12 < x < 2$

No test prep for this lesson.