2.10 Derivatives of tan x, cot x, sec x, and csc x

Calculus

Solutions

Practice

Find the derivative of each function.

1. $y = 5 - \csc x$

$y' = \csc x \cot x$

2. $h(x) = 2x\tan(x)$

$h'(x) = 2\tan x + 2x\sec^2 x$

3. $r = \frac{\sin\theta}{\theta}$

$\frac{dr}{d\theta} = \frac{\cos\theta \cdot \theta - \sin\theta}{\theta^2}$

4. $g(x) = \frac{\cot x}{x}$

$g'(x) = \frac{-\csc^2 x \cdot x - \cot x}{x^2}$

5. $f(x) = \frac{1}{2\cos x}$ $= \frac{1}{2}\sec x$

$f'(x) = \frac{1}{2}\sec x \tan x$

6. $y = 5x\sec x$

$\frac{dy}{dx} = 5\sec x + 5x\sec x\tan x$

Find the derivative at the given x-value. Show your work!

7. $f(x) = 3\tan x$ at $x = \frac{2\pi}{3}$.

$f'(x) = 3\sec^2 x$

$f'(x) = \frac{3}{\cos^2 x}$

$f'\left(\frac{2\pi}{3}\right) = \frac{3}{\left(-\frac{1}{2}\right)^2}$

$= 3 \cdot 4$

$= \boxed{12}$

8. $f(x) = 2\sec x$ at $x = \frac{\pi}{4}$.

$f'(x) = 2\sec x \tan x$

$f'(x) = \frac{2}{\sec x} \tan x$

$f'\left(\frac{\pi}{4}\right) = \frac{2}{\frac{\sqrt{2}}{2}} \cdot (1)$

$= 2 \cdot \frac{2}{\sqrt{2}}$

$= \frac{4}{\sqrt{2}}$

$= \boxed{2\sqrt{2}}$

9. $f(x) = x\cot x$ at $x = \frac{\pi}{6}$.

$f'(x) = 1 \cdot \cot x + x(-\csc^2 x)$

$f'(x) = \frac{\cos x}{\sin x} - \frac{x}{\sin^2 x}$

$f'\left(\frac{\pi}{6}\right) = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} - \frac{\frac{\pi}{6}}{\left(\frac{1}{2}\right)^2}$

$= \sqrt{3} - \frac{\pi}{6} \cdot 4$

$= \boxed{\sqrt{3} - \frac{2\pi}{3}}$

Estimate the derivative at the given x-value by using a calculator.

10. $f(x) = \sin^2\left(\frac{x}{5}\right)$ at $x = 1.8$.

$f'(1.8) = 0.1318$

11. $f(x) = \frac{\cot(x^2)}{2}$ at $x = -1$.

$f'(-1) = 1.412$

12. $f(x) = 3\sec(e^x)$ at $x = 2.5$.

$f'(2.5) = -15.9228$

Find the equations of both the normal line and the tangent line.

13. $y = \sec x$ at $x = \pi$

$y(\pi) = \frac{1}{\cos(\pi)} = -1 \leftarrow y_1$

$y' = \sec x \tan x$

$y' = \frac{1}{\cos\pi} \cdot \frac{\sin\pi}{\cos\pi} = 0 \leftarrow m$

$y + 1 = 0(x - \pi)$

Tangent: $y = -1$

Normal: $x = \pi$

14. $y = \tan x$ at $x = \frac{\pi}{3}$

$y\left(\frac{\pi}{3}\right) = \frac{\sin\pi/3}{\cos\pi/3} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

$y' = \sec^2 x$

$y'\left(\frac{\pi}{3}\right) = \frac{1}{\cos^2\left(\frac{\pi}{3}\right)} = \frac{1}{\left(\frac{1}{2}\right)^2} = 4$

Tangent: $y - \sqrt{3} = 4\left(x - \frac{\pi}{3}\right)$

Normal: $y - \sqrt{3} = -\frac{1}{4}\left(x - \frac{\pi}{3}\right)$

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Test Prep

Evaluate each limit.

15. $\lim_{h \to 0} \frac{\tan\left(\frac{\pi}{3}+h\right) - \tan\left(\frac{\pi}{3}\right)}{h} =$ Def. of Der.

$f(x) = \tan x$

$f'(x) = \sec^2 x$

$f'\left(\frac{\pi}{3}\right) = \sec^2\left(\frac{\pi}{3}\right) = \boxed{4}$

Same as problem #14

16. $\lim_{h \to 0} \frac{\sec\left(\frac{\pi}{6}+h\right) - \sec\left(\frac{\pi}{6}\right)}{h} =$

$f(x) = \sec x$

$f'(x) = \sec x \tan x$

$f'\left(\frac{\pi}{6}\right) = \frac{1}{\cos\left(\frac{\pi}{6}\right)} \cdot \frac{\sin\left(\frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right)}$

$= \frac{1}{\frac{\sqrt{3}}{2}} \cdot \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{4}{3}$

$\boxed{\frac{2}{3}}$