2.1 Average and Instantaneous Rate of Change

Notes

Write your questions and thoughts here!

Recall: Rate of Change

Average rate of change on the interval $[a, b]$ is represented by

Average rate of change from a **function**.
Find the average rate of change of $f(x) = \ln 3x$ over the interval $1 \leq x \leq 4$.

Average rate of change from a **table**.

x	0	2	7	30
$f(x)$	3	-2	5	7

Find the average rate of change over the interval $2 \leq x \leq 30$.

Average Rate of Change:
The following quotients express the average rate of change of a function over an interval.

$$\frac{f(a+h)-f(a)}{(a+h)-a} \quad \text{or} \quad \frac{f(x)-f(a)}{x-a}$$

This is also the of the line.

Instantaneous Rate of Change:
The following limits express the *instantaneous* rate of change of a function at $x = a$.

$$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} \quad \text{or} \quad \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

This is also the of the line.

Write your questions and thoughts here!

Find the instantaneous rate of change of $f(x) = x - x^2$ at $x = -1$.

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Identify the function we are working with. Then identify the x-value for the instantaneous rate of change (slope of the tangent line at a point).

1. $\lim_{h \to 0} \frac{5\ln\left(\frac{2}{4+h}\right) - 5\ln\left(\frac{1}{2}\right)}{h}$

Function: $f(x) =$

Instantaneous rate at $x =$

2. $\lim_{x \to \frac{\pi}{2}} \frac{\sin x - 1}{x - \frac{\pi}{2}}$

Function: $f(x) =$

Instantaneous rate at $x =$

2.1 Average and Instantaneous Rate of Change

Calculus

Practice

Find the average rate of change of each function on the given interval. Use appropriate units if necessary.

1. $f(x) = x^2 - 2$; $[-1, 3]$

2. $A(t) = 2^t$; $[2, 4]$
   t represents years
   A represents dollars

3. $h(m) = \tan(m) + 4$; $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$
   h represents hair
   m represents months

4. $a(x) = \ln x$ on the interval $2 \leq x \leq 7$.

5. $f(x) = \cos x$ on the interval $-1 \leq x \leq 0$.

Use the following table to find the average rate of change on the given interval.

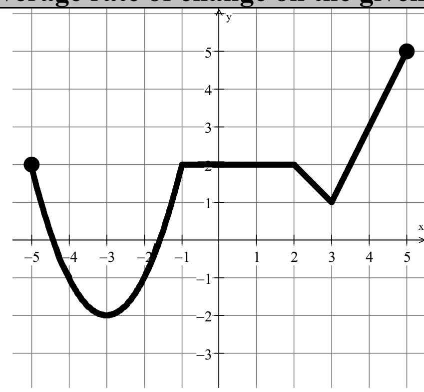
t (Minutes)	0	3	4	12	13
$s(t)$ (Feet)	-2	4	-7	5	10

6. $[3, 13]$

7. $0 \leq x \leq 12$

8. $[3, 4]$

Use the following graph to find the average rate of change on the given interval.

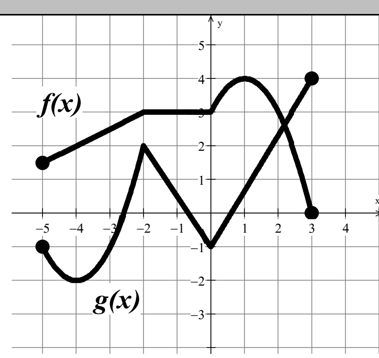


9. $-5 \leq x \leq -2$

10. $[-1, 5]$

11. $-4 \leq x \leq -2$

The graphs of f and g are given below. For each function, find the average rate of change on the given interval.



12. $h(x) = f(x) + g(x)$ on $[-4,3]$

13. $k(x) = f\big(g(x)\big)$ on $[-4,0]$

14. $w(x) = g\big(f(x)\big)$ on $[-2,3]$

Find the instantaneous rate of change of each function at the given x-value. Use the form $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$.

15. $f(x)=x^2-x$ at $x=-1$

16. $f(x)=\sqrt{x}$ at $x=5$

17. $f(x)=\frac{1}{x}$ at $x=2$

Find the instantaneous rate of change of each function at the given x-value. Use the form $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$.

18. $f(x)=2x^2+3x-4$ at $x=-3$

19. $f(x)=\sqrt{3x}$ at $x=7$

20. $f(x)=\frac{1}{5x}$ at $x=-2$

Each limit represents the instantaneous rate of change of a function. Identify the original function, and the x-value of the instantaneous rate of change.

21. $\lim_{x\to 7}\frac{\frac{1}{\sqrt{x^2-2x}}-\frac{1}{\sqrt{35}}}{x-7}$

Function: $f(x)=$

Instantaneous rate at $x=$

22. $\lim_{x\to -2}\frac{(3x-9x^2)+(42)}{x+2}$

Function: $f(x)=$

Instantaneous rate at $x=$

23. $\lim_{h\to 0}\frac{3\ln(2+h)-3\ln 2}{h}$

Function: $f(x)=$

Instantaneous rate at $x=$

24. $\lim_{h\to 0}\frac{3(1+h)^2-7(1+h)+1+(3)}{h}$

Function: $f(x)=$

Instantaneous rate at $x=$

25. $\lim_{x\to\frac{\pi}{2}}\frac{6x^2\sin x-\frac{3\pi^2}{2}}{x-\frac{\pi}{2}}$

Function: $f(x)=$

Instantaneous rate at $x=$

26. $\lim_{h\to 0}\frac{\log(2-4(h-5))-\log(22)}{h}$

Function: $f(x)=$

Instantaneous rate at $x=$

27. $\lim_{x\to 5}\frac{\frac{1}{\sqrt{3x}}-\frac{1}{\sqrt{15}}}{x-5}$

Function: $f(x)=$

Instantaneous rate at $x=$

28. $\lim_{h\to 0}\frac{e^{6(3+h)+1}-e^{19}}{h}$

Function: $f(x)=$

Instantaneous rate at $x=$

29. Let f be the function defined by $f(x) = \ln 7x$. The average rate of change of f over the interval $[2, a]$ is 41, where $a > 2$. Which of the following is an equation that could be used to find the value of a?

(A) $f(a) = 41$

(B) $f(a) - f(2) = 41$

(C) $\frac{f(a)-f(2)}{a-2} = 41$

(D) $\frac{f(a)+f(2)}{2} = 41$

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30. Find the average rate of change of $f(x) = \sin x \ln x$ on the interval $1 \leq x \leq a$.

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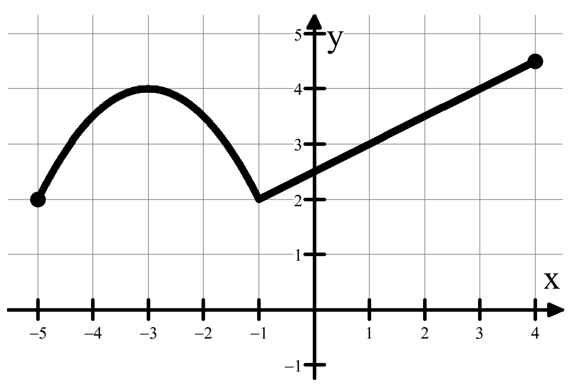
31. Today's school lunch was inappropriately thrown over the school fence by Mr. Kelly. For $0 \leq t \leq 90$, the amount of food remaining (assuming no animals eat it) is modeled by $F(t) = 544.311(0.907)^t$, where $F(t)$ is measured in grams and t is measured in days. Find the average rate of change of $F(t)$ over the interval $0 \leq t \leq 90$. Indicate units of measure.

32.



A continuous function f is shown above and defined on the closed interval $-5 \leq x \leq 4$. For how many values of b, $-5 < b < 4$, is the average rate of change of f on the interval $[b, 1]$ equal to 0? Give a reason for your answer.