2.1 Average and Instantaneous Rate of Change

Calculus

Solutions

Practice

Find the average rate of change of each function on the given interval. Use appropriate units if necessary.

1. $f(x) = x^2 - 2; \quad [-1, 3]$

$$\frac{f(3) - f(-1)}{3 - (-1)}$$

$$\frac{7 - (-1)}{4} = \boxed{2}$$

2. $A(t) = 2^t; \quad [2, 4]$
   t represents years
   A represents dollars

$$\frac{2^4 - 2^2}{4 - 2} = \frac{16 - 4}{2}$$

$\boxed{6 \text{ dollars / year}}$

3. $h(m) = \tan(m) + 4; \quad \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$
   h represents hair
   m represents months

$$\frac{(-1+4) - (1+4)}{3\pi/4 - \pi/4} = \frac{-2}{\pi/2}$$

$\boxed{-4/\pi \text{ hair/month}}$

4. $a(x) = \ln x$ on the interval $2 \leq x \leq 7$.

$$\frac{\ln 7 - \ln 2}{7 - 2}$$

$\boxed{0.251 \text{ rounded}}$

or

$\boxed{0.250 \text{ truncated}}$

5. $f(x) = \cos x$ on the interval $-1 \leq x \leq 0$.

$$\frac{\cos(0) - \cos(-1)}{0 - -1}$$

$\boxed{0.460 \text{ rounded}}$

$\boxed{0.459 \text{ truncated}}$

Use the following table to find the average rate of change on the given interval.

t (Minutes)	0	3	4	12	13
$s(t)$ (Feet)	-2	4	-7	5	10

6. $[3, 13]$

$$\frac{10-4}{13-3} = \frac{6}{10}$$

$\boxed{\tfrac{3}{5} \text{ feet/min}}$

7. $0 \le x \le 12$

$$\frac{5--2}{12-0} = \frac{7}{12}$$

$\boxed{\tfrac{7}{12} \text{ feet/min}}$

8. $[3, 4]$

$$\frac{-7-4}{4-3} = -11$$

$\boxed{-11 \text{ feet/min}}$

Use the following graph to find the average rate of change on the given interval.

9. $-5 \le x \le -2$

$$\frac{-1-2}{-2+5} = \frac{-3}{3}$$

$\boxed{-1}$

10. $[-1, 5]$

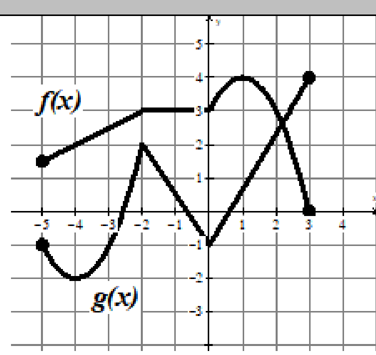
$$\frac{5-2}{5--1} = \boxed{\frac{1}{2}}$$

11. $-4 \le x \le -2$

$$\frac{-1--1}{-2--4} = \frac{0}{2}$$

$\boxed{0}$

The graphs of f and g are given below. For each function, find the average rate of change on the given interval.



12. $h(x) = f(x) + g(x)$ on $[-4,3]$

$$\frac{h(3)-h(-4)}{3--4}$$

$$\frac{4-0}{7} = \boxed{4/7}$$

13. $k(x) = f(g(x))$ on $[-4,0]$

$$\frac{k(0)-k(-4)}{0--4}$$

$$\frac{3-3}{4} = \boxed{0}$$

14. $w(x) = g(f(x))$ on $[-2,3]$

$$\frac{w(3)-w(-2)}{3--2}$$

$$\frac{-1-4}{5} = \boxed{-1}$$

Find the instantaneous rate of change of each function at the given x-value. Use the form $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$.

15. $f(x) = x^2 - x$ at $x = -1$

$$\frac{(-1+h)^2-(-1+h)-[(-1)^2-(-1)]}{h}$$

$$\lim_{h\to 0}\frac{(1-2h+h^2)+1-h-2}{h}$$

$$\lim_{h\to 0}\frac{-3h+h^2}{h}$$

$$\lim_{h\to 0}(-3+h) = \boxed{-3}$$

16. $f(x) = \sqrt{x}$ at $x = 5$

$$\lim_{h\to 0}\frac{\sqrt{5+h}-\sqrt{5}}{h}\cdot\frac{\sqrt{5+h}+\sqrt{5}}{\sqrt{5+h}+\sqrt{5}}$$

$$\lim_{h\to 0}\frac{5+h-5}{h(\sqrt{5+h}+\sqrt{5})}$$

$$\lim_{h\to 0}\frac{1}{\sqrt{5+h}+\sqrt{5}} = \boxed{\frac{1}{2\sqrt{5}}}$$

17. $f(x) = \frac{1}{x}$ at $x = 2$

$$\lim_{h\to 0}\frac{\frac{1}{2+h}-\frac{1}{2}}{h}$$

$$\lim_{h\to 0}\left(\frac{2-(2+h)}{2(2+h)}\right)\cdot\frac{1}{h}$$

$$\lim_{h\to 0}\frac{-1}{2(2+h)} = \boxed{-\frac{1}{4}}$$

Find the instantaneous rate of change of each function at the given x-value. Use the form $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$.

18. $f(x) = 2x^2 + 3x - 4$ at $x = -3$

$$\lim_{x\to -3}\frac{2x^2+3x-4-(18-9-4)}{x+3}$$

$$\lim_{x\to -3}\frac{2x^2+3x-9}{x+3}$$

$$\lim_{x\to -3}\frac{\cancel{(x+3)}(2x-3)}{\cancel{x+3}}$$

$$2(-3)-3 = \boxed{-9}$$

19. $f(x) = \sqrt{3x}$ at $x = 7$

$$\lim_{x\to 7}\frac{\sqrt{3x}-\sqrt{21}}{x-7}\cdot\frac{\sqrt{3x}+\sqrt{21}}{\sqrt{3x}+\sqrt{21}}$$

$$\lim_{x\to 7}\frac{3x-21}{(x-7)(\sqrt{3x}+\sqrt{21})}$$

$$\lim_{x\to 7}\frac{3}{\sqrt{3x}+\sqrt{21}}$$

$$\boxed{\frac{3}{2\sqrt{21}}}$$

20. $f(x) = \frac{1}{5x}$ at $x = -2$

$$\lim_{x\to -2}\frac{\frac{1}{5x}-\frac{1}{-10}}{x+2}$$

$$\lim_{x\to -2}\left(\frac{2}{10x}+\frac{x}{10x}\right)\left(\frac{1}{x+2}\right)$$

$$\lim_{x\to -2}\frac{1}{10x}$$

$$\boxed{-\frac{1}{20}}$$

Each limit represents the instantaneous rate of change of a function. Identify the original function, and the x-value of the instantaneous rate of change.

21. $\lim_{x\to 7}\frac{\frac{1}{\sqrt{x^2-2x}}-\frac{1}{\sqrt{35}}}{x-7}$

Function: $f(x) = \frac{1}{\sqrt{x^2-2x}}$

Instantaneous rate at $x =$ 7

22. $\lim_{x\to -2}\frac{(3x-9x^2)+(42)}{x+2}$

Function: $f(x) = 3x-9x^2$

Instantaneous rate at $x = -2$

23. $\lim_{h\to 0}\frac{3\ln(2+h)-3\ln 2}{h}$

Function: $f(x) = 3\ln x$

Instantaneous rate at $x = 2$

24. $\lim_{h\to 0}\frac{3(1+h)^2-7(1+h)+1+(3)}{h}$

Function: $f(x) = 3x^2-7x+1$

Instantaneous rate at $x =$ 1

25. $\lim_{x\to\frac{\pi}{2}}\frac{6x^2\sin x-\frac{3\pi^2}{2}}{x-\frac{\pi}{2}}$

Function: $f(x) = 6x^2\sin x$

Instantaneous rate at $x = \frac{\pi}{2}$

26. $\lim_{h\to 0}\frac{\log(2-4(h-5))-\log(22)}{h}$

Function: $f(x) = \log(2-4x)$

Instantaneous rate at $x = -5$

27. $\lim_{x\to 5}\frac{\frac{1}{\sqrt{3x}}-\frac{1}{\sqrt{15}}}{x-5}$

Function: $f(x) = \frac{1}{\sqrt{3x}}$

Instantaneous rate at $x =$ 5

28. $\lim_{h\to 0}\frac{e^{6(3+h)+1}-e^{19}}{h}$

Function: $f(x) = e^{6x+1}$

Instantaneous rate at $x = 3$

29. Let f be the function defined by $f(x) = \ln 7x$. The average rate of change of f over the interval $[2, a]$ is 41, where $a > 2$. Which of the following is an equation that could be used to find the value of a?

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(A) $f(a) = 41$

(B) $f(a) - f(2) = 41$

(C) $\frac{f(a)-f(2)}{a-2} = 41$

(D) $\frac{f(a)+f(2)}{2} = 41$

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30. Find the average rate of change of $f(x) = \sin x \ln x$ on the interval $1 \le x \le a$.

$$\frac{\sin a \ln a - \sin(1)\ln 1}{a-1} = \frac{\sin(a)\ln(a)}{a-1}$$

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31. Today's school lunch was inappropriately thrown over the school fence by Mr. Kelly. For $0 \le t \le 90$, the amount of food remaining (assuming no animals eat it) is modeled by $F(t) = 544.311(0.907)^t$, where $F(t)$ is measured in grams and t is measured in days. Find the average rate of change of $F(t)$ over the interval $0 \le t \le 90$. Indicate units of measure.

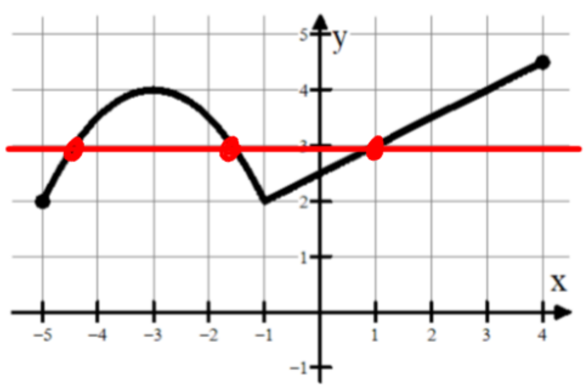
$$\frac{F(90)-F(0)}{90-0} \approx$$

-6.047 grams per day rounded

or

-6.046 grams per day truncated

32.



A continuous function f is shown above and defined on the closed interval $-5 < x < 4$. For how many values of b, $-5 < b < 4$, is the average rate of change of f on the interval $[b, 1]$ equal to 0? Give a reason for your answer.

Two. Avg rate of change = 0 means we have a horizontal line (slope of zero).