Calculus

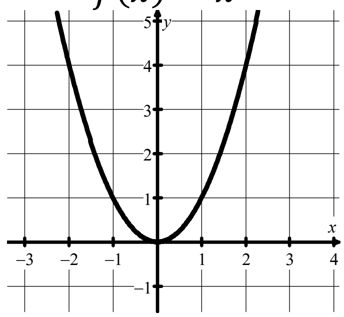
# 2.2 Defining the Derivative

**Notes**

Write your questions and thoughts here!

This ***derivative*** is an expression that calculates the instantaneous rate of change (slope of the tangent line) of a function at any given $x$-value. In other words, it gives us the slope of the function at a point!

$f(x) = x^2$




$f'(x) = 2x$

$f'(1) =$

$f'(2) =$

$f'(-2) =$

1. If $f'(x) = \frac{5}{x} - x$, find $f'(2)$ and explain the meaning.

2. If $f(x)$ represents how many meters you have run and $x$ represents the minutes, describe in full sentences the following:

| $f(8) = 1{,}500$ | $f'(3) = 161$ |
|---|---|

**<u>Notation for the Derivative:</u>**

| <u>Lagrange</u> | <u>Leibniz</u> |
|---|---|

**<u>Defintion of the Derivative:</u>**

This limit gives an expression that calculates the instantaneous rate of change (slope of the tangent line) of $f(x)$ at any given $x$-value.

Write your questions and thoughts here!

## Find the derivative using the Definition of the Derivative (limits).

3. $f(x) = 2x^2 - 7x + 1$

4. $y = \frac{1}{x^2}$

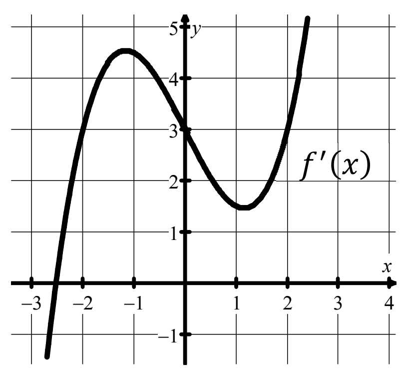
## Equation of the Tangent Line:

The line tangent to the curve of $f(x)$ at $x = a$ can be represented in point-slope form:

$$y - y_1 = m(x - x_1)$$

5. If we know $h(5) = -2$, and the derivative of $h$ is given by $h'(x) = \frac{x^3-2}{x}$, write an equation for the line tangent to the graph of $h$ at $x = 5$.

6. The graph of $f'(x)$, the derivative of $f$, is shown at the right. If $f(2) = 7$, write an equation of the line tangent to the graph of $f$ at $(2, 7)$.



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Calculus

**Practice**

**Find the derivative using limits. If the equation is given as $y =$, use Leibniz Notation: $\frac{dy}{dx}$. If the equation is given as $f(x) =$, use Lagrange Notation: $f'(x)$. WRITE SMALL!!**

1. $f(x) = 7 - 6x$

2. $y = 5x^2 - x$

3. $y = \sqrt{5x + 2}$

4. $f(x) = \frac{1}{x-2}$

**For each problem, use the information given to identify the meaning of the two equations in the context of the problem. Write in full sentences!**

5. $C$ is the number of championships Sully has won while coaching basketball. $t$ is the number of years since 2002 for the function $C(t)$.
   $C(12) = 3$ and $C'(12) = 0.4$

6. $d$ is the distance (in miles) from home when you walk to school. $h$ is the number of hours since 7:00 a.m. for the function $d(h)$.
   $d(0.5) = 1.2$ and $d'(0.5) = -11$

7. $W$ is the number of cartoon shows Mr. Kelly watches every week. $x$ is the number of children Mr. Kelly has for the function $W(x)$.
$W(7) = 25$ and $W'(7) = 3$

8. $g$ is the number of gray hairs on Mr. Brust's head. $x$ is the number of students in his 4th period.
$g(26) = 501$ and $g'(15) = 130$

**For each problem, create an equation of the tangent line of $f$ at the given point. Leave in point-slope.**

9. $f(7) = 5$ and $f'(7) = -2$

10. $f(-2) = 3$ and $f'(-2) = 4$

11. $f(x) = 3x^2 + 2x$;
$f'(x) = 6x + 2$; $x = -2$

12. $f(x) = 10\sqrt{6x+1}$;
$f'(x) = \frac{30}{\sqrt{6x+1}}$; $x = 4$

13. $f(x) = \cos 2x$;
$f'(x) = -2\sin 2x$; $x = \frac{\pi}{4}$

14. $f(x) = \tan x$;
$f'(x) = \sec^2 x$; $x = \frac{\pi}{3}$

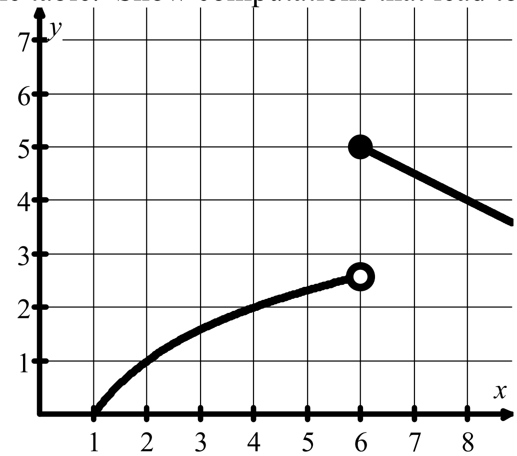
## 2.2 Defining the Derivative

**Test Prep**

15. Let $f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$. For what value of $x$ does $f(x) = 4$?

(A) $-4$ (B) $-1$ (C) $1$ (D) $2$ (E) $4$

16. The graph of the function $f$, along with a table of values, are shown below. Approximate the value of $f'(5.5)$ using data from the table. Show computations that lead to your answer.




| $x$ | 4.5 | 5 | 5.5 | 6 | 6.5 | 7 |
|---|---|---|---|---|---|---|
| $f(x)$ | 2.169 | 2.321 | 2.459 | 5 | 4.5 | 4 |

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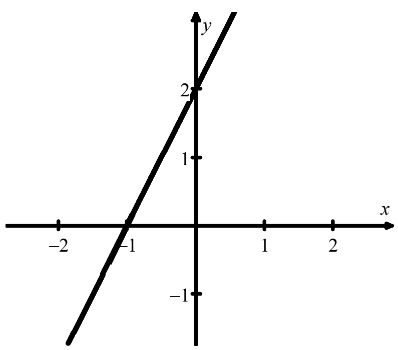
17. The figure below shows the graph of the line tangent to the graph of $f$ at $x = 0$.




Of the following, which must be true?

(A) $f'(0) = -f(0)$

(B) $f'(0) = f(0)$

(C) $f'(0) > f(0)$

(D) $f'(0) < f(0)$