2.2 Defining the Derivative

Calculus

Practice

Find the derivative using limits. If the equation is given as $y =$, use Leibniz Notation: $\frac{dy}{dx}$. If the equation is given as $f(x) =$, use Lagrange Notation: $f'(x)$. WRITE SMALL!!

1. $f(x) = 7 - 6x$

$$f'(x) = \lim_{h\to 0} \frac{7-6(x+h)-(7-6x)}{h}$$

$$= \lim_{h\to 0} \frac{7-6x-6h-7+6x}{h}$$

$$= \lim_{h\to 0} \frac{-6h}{h}$$

$$\boxed{f'(x) = -6}$$

2. $y = 5x^2 - x$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{5(x+h)^2-(x+h)-(5x^2-x)}{h}$$

$$= \lim_{h\to 0} \frac{5(x^2+2hx+h^2)-x-h-5x^2+x}{h}$$

$$= \lim_{h\to 0} \frac{10hx+5h^2-h}{h} = \frac{h(10x+5h-1)}{h}$$

$$= \lim_{h\to 0} 10x+5h-1$$

$$\boxed{\frac{dy}{dx} = 10x-1}$$

3. $y = \sqrt{5x+2}$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{\sqrt{5(x+h)+2}-\sqrt{5x+2}}{h} \cdot \frac{\sqrt{5(x+h)+2}+\sqrt{5x+2}}{\sqrt{5(x+h)+2}+\sqrt{5x+2}}$$

$$= \lim_{h\to 0} \frac{5(x+h)+2-(5x+2)}{h(\sqrt{5(x+h)+2}+\sqrt{5x+2})}$$

$$= \lim_{h\to 0} \frac{5x+5h+2-5x-2}{h(\sqrt{5(x+h)+2}+\sqrt{5x+2})}$$

$$= \lim_{h\to 0} \frac{5\cancel{h}}{\cancel{h}(\sqrt{5(x+\underset{0}{h})+2}+\sqrt{5x+2})}$$

$$\boxed{\frac{dy}{dx} = \frac{5}{2\sqrt{5x+2}}}$$

4. $f(x) = \frac{1}{x-2}$

$$f'(x) = \lim_{h\to 0} \frac{\frac{1}{(x+h)-2} - \frac{1}{x-2}}{h}$$

$$= \lim_{h\to 0} \left(\frac{x-2}{(x+h-2)(x-2)} - \frac{x+h-2}{(x+h-2)(x-2)}\right) \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \left(\frac{-h}{(x+h-2)(x-2)}\right) \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{-1}{(x+h-2)(x-2)}$$

$$\boxed{f'(x) = -\frac{1}{(x-2)^2}}$$

For each problem, use the information given to identify the meaning of the two equations in the context of the problem. Write in full sentences!

5. C is the number of championships Sully has won while coaching basketball. t is the number of years since 2002 for the function $C(t)$.
$C(12) = 3$ and $C'(12) = 0.4$

By 2014, Sully won 3 championships.

In 2014, Sully is winning 0.4 championships per year.

6. d is the distance (in miles) from home when you walk to school. h is the number of hours since 7:00 a.m. for the function $d(h)$.
$d(0.5) = 1.2$ and $d'(0.5) = -11$

At 7:30, I am 1.2 miles from home.

At 7:30, I am going back home at 11 miles per hour.

7. W is the number of cartoon shows Mr. Kelly watches every week. x is the number of children Mr. Kelly has for the function $W(x)$.
$W(7) = 25$ and $W'(7) = 3$

If Mr. Kelly has 7 kids, he watches 25 cartoons each week.

If he has 7 kids, the rate of watching cartoons is increasing by 3 per week.

8. g is the number of gray hairs on Mr. Brust's head. x is the number of students in his 4th period.
$g(26) = 501$ and $g'(15) = 130$

With 26 kids in his 4th period, Mr. Brust has 501 gray hairs.

With 15 kids in his 4th period, Mr. Brust is gaining 130 gray hairs per kid.

For each problem, create an equation of the tangent line of f at the given point. Leave in point-slope.

9. $f(7) = 5$ and $f'(7) = -2$

$$\boxed{y - 5 = -2(x - 7)}$$

10. $f(-2) = 3$ and $f'(-2) = 4$

$$\boxed{y - 3 = 4(x + 2)}$$

11. $f(x) = 3x^2 + 2x$;
$f'(x) = 6x + 2$; $x = -2$

$f(-2) = 12 - 4 = 8$
$f'(-2) = -12 + 2 = -10$

$$\boxed{y - 8 = -10(x + 2)}$$

12. $f(x) = 10\sqrt{6x+1}$;
$f'(x) = \frac{30}{\sqrt{6x+1}}$; $x = 4$

$f(4) = 10\sqrt{25} = 50$
$f'(4) = \frac{30}{\sqrt{25}} = 6$

$$\boxed{y - 50 = 6(x - 4)}$$

13. $f(x) = \cos 2x$;
$f'(x) = -2\sin 2x$; $x = \frac{\pi}{4}$

$f(\frac{\pi}{4}) = \cos(\frac{\pi}{2}) = 0$
$f'(\frac{\pi}{4}) = -2\sin(\frac{\pi}{2}) = -2$

$$\boxed{y = -2(x - \tfrac{\pi}{4})}$$

14. $f(x) = \tan x$;
$f'(x) = \sec^2 x$; $x = \frac{\pi}{3}$

$f(\frac{\pi}{4}) = \tan\frac{\pi}{3} = \sqrt{3}$
$f'(\frac{\pi}{4}) = \frac{1}{\cos^2(\frac{\pi}{3})} = \frac{1}{(\frac{1}{2})^2} = 4$

$$\boxed{y - \sqrt{3} = 4(x - \tfrac{\pi}{3})}$$

2.2 Defining the Derivative

Test Prep

$f(x+h) - f(x)$

15. Let $f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$. For what value of x does $f(x) = 4$?

$f(x) = x^2$ $\qquad$ $x^2 = 4$

(A) -4 (B) -1 (C) 1 (D) 2 (E) 4

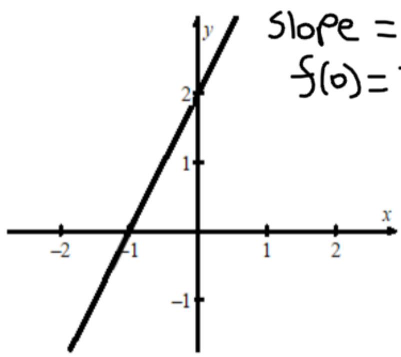
16. The graph of the function f, along with a table of values, are shown below. Approximate the value of $f'(5.5)$ using data from the table. Show computations that lead to your answer.

x	4.5	5	5.5	6	6.5	7
$f(x)$	2.169	2.321	2.459	5	4.5	4

$$\frac{2.459-2.321}{5.5-5} = \boxed{0.276}$$

---

17. The figure below shows the graph of the line tangent to the graph of f at $x = 0$.



Slope = 2
$f(0)=2$

Of the following, which must be true?

(A) $f'(0) = -f(0)$

(B) $f'(0) = f(0)$

(C) $f'(0) > f(0)$

(D) $f'(0) < f(0)$