2.3 Estimating Derivatives

Calculus

Solutions

Estimate the derivative at the given point by using a calculator.

1. $f(x) = x\sqrt{2-x}$; find $f'(-10)$. 4.907	2. $f(x) = \sec(5x)$; find $f'(2)$. -3.863 or -3.864	3. $f(x) = \ln(\sqrt{x})$; find $f'(1)$. 0.5
4. $f(x) = e^{\frac{x}{3}}$; find $f'(4)$. 1.264 or 1.265	5. $f(x) = \tan(\sin x)$; find $f'(-3)$. -1.009 or -1.010	6. $f(x) = 2^{\ln(x)}$; find $f'(2)$. 0.560
7. The model $f(t) = \dfrac{x}{\cos x}$ measures the height of bird in meters where t is seconds. Find $f'(2)$. 8.098 m/sec	8. The model $f(t) = \sin^2(t)$ measures the depth of a submarine measured in feet where t is minutes. Find $f'(12.5)$. -0.132 ft/min	9. The model $f(t) = \sqrt{x} - \dfrac{1}{x-1}$ measures the number of stocks sold where t is days. Find $f'(12)$ 0.1526 Stocks/day

For each function, write the equation of the tangent line at the given value of x.

10. $f(x) = \dfrac{\ln 2x}{4x}$ at $x = 1$. $y - 0.173 = 0.0767(x-1)$	11. $f(x) = \cos(\tan(x))$ at $x = 2$. $y + 0.576 = 4.7188(x-2)$
12. $f(x) = \dfrac{x^4}{\sqrt{x}}$ at $x = 3$. $y - 46.765 = 54.5596(x-3)$	13. $f(x) = x^2 \sin\left(\frac{1}{x}\right)$ at $x = 7$. $y - 6.976 = 1.003(x-7)$

Use the tables to estimate the value of the derivative at the given point. Indicate units of measures.

14.

x Hours	1	3	4	7	9
$v(x)$ visitors	120	476	595	807	902

a. $f'(8)$

$\dfrac{902 - 807}{9 - 7} = \boxed{47.5}$ Visitors/hr

b. $f'(3.5)$

$\dfrac{595 - 476}{4 - 3} = \boxed{119}$ visitors/hr

15.

x cm	11	23	26	32	45
$T(x)$ °C	71	51	40	36	10

a. $f'(17)$

$$\frac{51-71}{23-11} \approx \boxed{-1.667 \text{ °C per cm}}$$

b. $f'(24.5)$

$$\frac{40-51}{26-23} \approx \boxed{-3.667 \text{ °C per cm}}$$

16.

t years	0	3	7	15	20
$s(t)$ Students per year	5	20	7	-2	-4

a. $f'(1.5)$

$$\frac{20-5}{3-0} = \boxed{5 \text{ students/year}^2}$$

b. $f'(11)$

$$\frac{-2-7}{15-7} = \boxed{-1.125 \text{ Students/yr}^2}$$

17.

t Days	5	13	45	50	70
$p(t)$ Pages per day	51	20	21	36	58

a. $f'(47.5)$

$$\frac{36-21}{50-45} = \boxed{3 \text{ pgs/day}^2}$$

b. $f'(9)$

$$\frac{20-51}{13-5} = \boxed{-3.875 \text{ pgs/day}^2}$$

18.

x seconds	10	30	45	65	100
$w(x)$ Gallons per second	1005	790	786	434	209

a. $f'(20)$

$$\frac{790-1005}{30-10} = \boxed{-10.75 \text{ gallons/sec}^2}$$

b. $f'(82.5)$

$$\frac{209-434}{100-65} \approx \boxed{-6.4285 \text{ gallons/sec}^2}$$

19.

x Carries	3	12	15	21	30
$f(x)$ yards	15	107	98	150	272

a. $f'(25.5)$

$$\frac{272-150}{30-21} \approx \boxed{13.5555 \text{ yards per carry}}$$

b. $f'(13.5)$

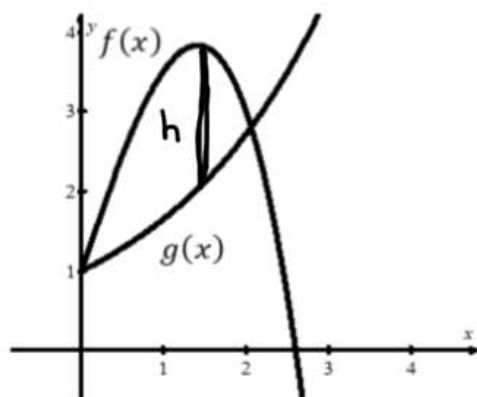$$\frac{98-107}{15-12} = \boxed{-3 \text{ yards per carry}}$$

20. Let f and g be the functions defined by $f(x) = -\frac{1}{2}x^3 + 3x + 1$ and $g(x) = e^{\frac{x}{2}}$. Let h be the vertical distance between the graphs of f and g for $0 \le x \le 2$. Find the rate at which h changes with respect to x when $x = 1.5$.

$$h(x) = f(x) - g(x)$$
$$h'(1.5) = f'(1.5) - g'(1.5)$$

$$\boxed{h'(1.5) = -1.4335}$$

21. The graph of $y = 3 - e^{5x}$ crosses the x-axis at one point. What is the slope of the graph at this point

$$3 - e^{5x} = 0$$

← Don't round!

$$x \simeq 0.2197225 = a$$

$$\boxed{y'(a) = -15}$$

22. Given the function $g(x) = x^3 - e^x - \sin x$, which of the following values of x has a tangent line with the greatest slope?

Compare the following

$$g'(-3) \qquad g'(-1) \qquad g'(0) \qquad g'(1) \qquad g'(3)$$

(A) $x = -3$ (B) $x = -1$ (C) $x = 0$ (D) $x = 1$ (E) $x = 3$